CHAPTER ELEVEN

Complex Dynamical Systems in Social and Personality Psychology

Theory, Modeling, and Analysis

MICHAEL J. RICHARDSON, RICK DALE, AND KERRY L. MARSH

All social processes fundamentally involve change in time: Judgments materialize quickly over milliseconds or seconds, conversations flow over minutes, and relationships evolve across even longer time scales. Put simply, social systems are dynamical systems. The word "dynamical" simply means time-evolving and thus a *dynamical system* is simply a system whose behavior evolves or changes over time. Proposing that social processes are dynamical is not new and has a long history in social psychology (e.g., Asch, 1952; Lewin, 1936; Mead, 1934). Moreover, most researchers in social-personality psychology would agree that social processes and behavior are dynamical and change over time. Traditionally, however, social-personality psychology, like experimental psychology in general, has focused on summary statistics, which aggregate over time, such as in the form of magnitudes (e.g., behavioral frequencies, emotional intensities, and so on). This traditional approach is rooted in the linear statistical methods developed by Fisher and others (Meehl, 1978), and is aimed at detecting whether treatments or manipulations, on the whole, affect the outcome of some measured behavioral state or variable. Behavioral change is therefore conceptualized as the difference between static measures and is modeled by covarying responses on such measures. Unfortunately, this traditional approach merely describes behavioral change; it does not capture true time evolution and so is not always optimal for understanding the process by which behavioral change occurs. To make progress in our understanding of psychological *change* and *process*, therefore, researchers need to consider adopting new tools and methodological concepts, namely those of dynamical systems.

The scientific study of dynamical systems is concerned with understanding, modeling, and predicting the ways in which the behavior of a system changes over time. As a formal approach, it has a long history in applied mathematics and physics, and has been used extensively to understand and model the behavior of many different types of physical systems, such as the motion (position and velocity) of planets, mass-spring systems, swinging pendulums, and self-sustained oscillators. In the last few decades, however, an increasing number of researchers have begun to investigate and understand the dynamic behavior of more *complex* biological, cognitive, and social systems, using the concepts and tools of dynamical systems.

The term "complexity" refers to the fact that most biological, cognitive, and social systems typically exhibit behavior that is nonlinear and involves a large number of interacting elements or components. Historically, it is the nonlinearity of complex dynamical systems that has largely hindered research on such systems, in that the numerical techniques that enable one to uncover the dynamics of nonlinear and complex dynamical systems involve an extensive number of computational processes that are impossible to perform without modern computers. This is true for both abstract nonlinear dynamical models (covered in the second section of this chapter) and for the analysis of behavioral data (discussed in the third section). These days, of course, these difficulties of computation no longer exist, and researchers can formulate and analyze many nonlinear and complex dynamical systems quite easily. Indeed, the fields of nonlinear dynamics and complex systems, as well as our theoretical understanding of such systems, have grown in parallel with increases in the availability and computational power of modern computers.

Advances in the modeling and analysis of complex dynamical systems have also led to the steady rise of dynamical systems in social-personality psychology. Vallacher, Read, and Nowak (2002) offer an excellent review of this rise and describe how dynamical theory and models can be employed to characterize an array of socially relevant behaviors, such as attitude change, social judgment, and self-perception (for reviews, see Nowak & Vallacher, 1998; Vallacher & Nowak, 1994). This interest in dynamics and the dynamical systems approach has continued to expand in social-personality psychology, with numerous researchers embracing a dynamical understanding of mood (e.g., Gottschalk, Bauer, & Whybrow, 1995; Schuldberg & Gottlieb, 2002), personality expression (Brown & Moskowitrz, 1998; Mischel & Shoda, 1995), conformity (Tesser & Achee, 1994), romantic relationships (Gottman, Swanson, & Swanson, 2002), and person perception and construal (Freeman & Ambady, 2011). Along with more recent advances in fractal methods (Correll, 2008; Delignières, Fortes, & Ninot, 2004), decision dynamics (Freeman, Dale, & Farmer, 2011), cognitive dynamics (Spivey, 2007), coordination dynamics (Kelso, 1995; Schmidt & Richardson, 2008), and the behavioral dynamics of perception and action (Warren, 2006), the concepts and tools that have been developed for understanding complex dynamical systems appear to provide a promising new method for understanding social behavior and cognition.

Here we provide an introductory overview of the dynamical systems approach and how the theoretical concepts, modeling techniques, and analysis tools used to investigate complex dynamical systems can be used to understand social behaviors that emerge and change over time. It is by no means a comprehensive review of complex dynamical systems and the dynamical systems approach. Rather, the chapter is aimed at providing the reader with the knowledge needed to seek out more detailed discussions elsewhere. Throughout the chapter we include key references that provide a deeper and more detailed understanding of the concepts and issues discussed.

The chapter is divided into three main sections. The first section covers the basic concepts of dynamical systems theory, including their implications for understanding human and social behavior, and how complex ordered behavior emerges from the nonlinear interactions that exist between the components of a behavioral system. The second section covers the basic forms and mathematical properties of dynamical systems models and how dynamical modeling can provide insights about the organization or reorganization of stable (or unstable) human and social behavior. The final section addresses the basic aspects of dynamical systems analysis and focuses on a number of analysis techniques that can be employed to uncover the dynamics of a behavioral system from time-series recordings.

DYNAMICAL SYSTEMS THEORY

The keystone concept throughout this section (and the entire chapter) is that the behavior of a complex dynamical system can arise in a self-organized manner from the free interplay of components and properties of the system. To unpack what this means, we begin by briefly defining what a complex dynamical system is and then go on to describe the abstract properties that underlie the behaviors that complex dynamical systems exhibit.

What Is a Complex Dynamical System?

The term "complex dynamical system" lacks a consensus definition, but many researchers agree that complex dynamical systems exhibit three key characteristics (see Gallagher & Appenzeller, 1999, and articles therein, for a discussion). First, they consist of a large number of interacting components or agents. This may be said about the behavior of an individual person, as in the interacting constraints that drive social perceptions and decisions (e.g., Read & Miller, 2002; Smith, 1996). It may also be said about the behavior of groups of human beings, such as in dyadic conversation (Buder, 1991) or small work-teams (Arrow, 1997). A second property is that these systems exhibit emergence: Their collective behavior can be difficult to anticipate from knowledge of the individual components that make up the system, and exhibit some coherent pattern or even, in some cases, apparent purposiveness. For example, a person's social judgment may be a nonobvious consequence of the interaction among an array of informational constraints (e.g., Freeman & Ambady, 2011), and group behaviors may be a nonobvious outcome of interactions among group members (Arrow, 1997). Third, this emergent behavior is self-organized and does not result from a central or external controlling component process or agent. Although all three characteristics are necessary for a system to be considered complex, the appearance of emergent behavior that results from self-organization is the most distinguishing feature of complex systems (Boccara, 2003). Accordingly, we turn next to the topic of self-organization.

Self-Organization

The term "self-organization" is used to refer to behavioral patterns that emerge from the interactions that bind the components of a system (social or otherwise) into a collective, synergistic system, while not being dictated a priori by a centralized controller. As mentioned earlier, self-organization is synonymous with complex systems, and interestingly enough, the most widely used examples of a self-organized and emergent behavioral process are social examples, in particular the coordinated activities of social insects such as ants. Take a colony of harvester ants, for example, in which different members of the ant colony perform one of several different tasks: foraging, patrolling, nest maintenance, or midden work (clearing up debris). Individual ants do not perform the same task all the time, but transition between the different modes of behavior as the need arises, with the appropriate number of ants (workers) engaged in a particular task at any given time. That is, patrollers become foragers, foragers become nest maintenance workers, and midden workers becoming patrollers, and so forth, based on current conditions (e.g., Gordon, 2007). Task allocation, however, is not achieved by a centralized controller or leader. The queen does not decide who does what, nor does any other ant. In fact, it would be impossible for any ant to oversee the entire colony. Rather, individual ants can only detect local tactile and chemical information, with the coordinated behavior of the colony emerging from the local interactions that constrain the tasks an individual ant should perform (Boccara, 2003).

The harvester ant colony highlights how coordinated social behavior can result spontaneously from the physical and informational interactions of perceiving-acting agents. The nest-building behavior of other social insects, such as termites, bees, and wasps, is similarly self-organized. So too is the coordinated behavior of schools of fish, flocks of birds, and herds of ungulate mammals: No individual animal has precise control over the direction or behavior of the group (e.g., Camazine, Deneubourg, Franks, Sneyd, Theraulaz, & Bonabeau, 2001; Theraulaz & Bonabeau, 1995).

Though it is certainly different from that of humans in numerable respects, the self-organization that ants and other social animals promote is directly relevant to, and sometimes motivates, the study of human social groups (Arrow, 1997). For instance, the coordinated activity observed between pedestrians walking down a crowded sidewalk is the result of self-organizing dynamics (Sumpter, 2010). Goldstone and colleagues have demonstrated that the human-path systems that are created between regularly visited destinations (say, buildings around a university campus) emerge in a self-organized manner and are often mutually advantageous to the members of the group that created them. Task subroles and divisions of labor can also emerge and be spontaneously adopted by individuals during joint action or a social cognitive task (Eguíluz, Zimmermann, Cela-Conde, & San Miguel, 2005; Goldstone & Gureckis, 2009; Richardson, Marsh, & Baron, 2007; Theiner, Allen, & Goldstone, 2010). Even group dynamics and performance can be self-organized by means of integrative complexity, whereby the individual ideas and opinions of group members emerge and become dynamically integrated over time (Cummings, Schlosser, & Arrow, 1996).

Soft-Assembly

The kinds of self-organized dynamical systems described in the preceding section, such as social insects and sometimes groups of people, are temporary organizational structures that are put together in a fluid and flexible manner. In the case of the harvester ants, it does not matter which particular ant does which particular job; each ant is capable of taking up any job at any point in time. Though obviously different in important ways, humans also engage in modes of behavior flexibly throughout the course of a single day (see Iberall & McCulloch, 1969 for classic a discussion; see Isenhower, Richardson, Marsh, Carello, & Baron, 2010 for a task example). Moreover, although it seems individuals in a group or social network are in complete control of their own behavior and are consciously aware of their acts (and could verbalize them if asked), we know that such centralized, conscious control is often an illusion: Classic research in social psychology suggests that individuals can be unaware of the "true" reasons for their actions (Nisbett & Wilson, 1977). Indeed, the coordinated behavior and intentions of socially situated individuals can be constrained and self-organized by environmental and situational constraints of which we are not aware, with the unfolding dynamics of human behavior reflecting a mutuality of responsiveness between individuals and the context in which they are embedded (Reis, 2008; Richardson, Marsh, & Schmidt, 2010;

Semin & Smith 2008; Thelen & Smith, 1994). Understanding and predicting the time-evolving behavior of an individual or social system is therefore not only dependent on identifying the environmental factors or agents that make up the behavioral system in question, but also on the relevant interagent and agent-environment interactions that shape behavior. This implies that the dynamical behavior of cognitive and social systems is highly context dependent.

Dynamical systems that exhibit this kind of emergent, context-dependent behavior are often referred to as *softly assembled* or *soft-molded* systems, in that the behavioral system reflects a temporary coalition of coordinated entities, components, or factors. The term "synergy" is sometimes used to refer to softly assembled systems – a functional grouping of structural elements (molecules, genes, neurons, muscles, limbs, individuals, etc.) that are temporarily constrained to act as a single coherent unit (Kelso, 2009). In contrast, most nonbiological systems or machines are hard-molded systems. A pendulum clock, for example, is a hard-molded dynamical system, in that it is composed of a series of components (parts), each of which plays a specific, predetermined, and unchanging role in shaping the motion of the clock's pendulum over time. Rigidly organized factory assembly lines, organizations, or military structures could also be considered intuitive examples of hard-molded machine-like social systems. But even within rigid social structures one finds differing degrees of fluidity and soft-assembly, both within and across different levels of organization. One could argue that this soft-assembly, present to some degree, is a necessary requirement for the ongoing existence and success of such social systems and organizations (e.g., Dooley, 1994; Guastello, 2002).

Interaction-Dominant Dynamics

The key property of softly assembled systems is that they exhibit interaction-dominant dynamics, as opposed to component-dominant dynamics. For component-dominant dynamical systems, system behavior is the product of a rigidly delineated architecture of system modules, component elements, or agents, each with predetermined functions (i.e., the pendulum clock or a factory assembly line). As noted earlier, however, for softly assembled interaction-dominant dynamical systems, system behavior is the result of *interactions between* system components, agents, and situational factors, with these intercomponent and interagent interactions altering the dynamics of the component elements, situational factors, and agents themselves (Anderson, Richardson, & Chemero, 2012; Kello, Beltz, Holden, & Van Orden, 2007; Van Orden, Kloos, & Wallot, 2011).

Thus, if one were to examine the relationship between any two levels of an interaction-dominant dynamical system, one would observe that elements or agents at the lower level of the system modulate the macroscopic order of the higher level and at the same time are structured by the macroscopic order of the system. For example, the individuals (micro-level) within a cultural system (macro-level) modulate the behavioral order of the cultural system, with the dynamical organization of the cultural system in turn (and at the same time) modulating and structuring the behavior of the individuals within it. Accordingly, for interaction-dominant systems, it is difficult – and often impossible – to assign precise causal roles to particular components, factors, agents, or system levels. Of particular significance for the study of cognitive and social behavior is the implication that one cannot hope to appropriately understand behavioral organization by attempting to study systems components or agents in isolation. For that reason, researchers who have adopted the complex dynamical approach to social phenomena have started to conceptualize many domains of human behavior, from language acquisition (Van Geert, 1991) to group dynamics (Arrow, McGrath, & Berdahl, 2000), as being guided and self-organized by the dynamic interaction of many constraints, factors, and processes.

Nonlinearity

A nonlinear system is one in which the system's output is not directly proportional to the input, as opposed to a linear system in which the output can be simply represented as a weighted sum of input components. Complex dynamical systems, most notably biological and social systems, are nonlinear in this sense. Our attraction to another individual or our self-concept, for instance, may not correspond to a mere average or sum of positive and negative attributes (Rinaldi & Gragnani, 1998; Sprott, 2004), but rather some form of multiplicative combination of attributes and situational factors that results in an attraction or self-concept that is more (or less) than the sum of its parts. The consequence of this principle for understanding human behavior and social systems is equivalent to the consequence of a system exhibiting interaction-dominant dynamics – a system's behavior cannot be reduced to a set of component dominant factors that interact in a simple linear fashion (Van Orden, Holden & Turvey, 2003). Thus, in the context of complex dynamical systems, the term "nonlinear" also means something more than multiplicative. More generally, it is used to refer to the non-decomposability of a complex dynamic system, whereby a nonlinear dynamical system is a system whose macroscopic behavioral order results from the complex interactions of micro-scale components, but via processes of circular causality, also modifies the interactions between micro-scale components as well as the behavior of the components themselves.

On the one hand, the nonlinearity of complex dynamical systems makes them much more difficult to understand. In fact, the effects of nonlinear processes are typically not known or cannot be known ahead of time. On the other hand, it is only because complex dynamical systems are nonlinear that they can exhibit complex emergent behavior. It is for these reasons that an increasing number of researchers and theorists consider human behavior and social processes to be predominantly nonlinear (see e.g., Guastello, Koopmans, & Pincus, 2009; Kelso, 1995; Vallacher & Nowak, 1994; Nowak & Vallacher, 1998 for edited collections and reviews). In fact, a defining feature of human behavior is that it is often unpredictable. It is this aspect of human behavior that makes the science of social-personality psychology, as well as the many other fields of psychology (perceptual, cognitive, clinical, etc.), so difficult and at the same time so intriguing. In addition, although the word "random" is often used in everyday speech when referring to the unpredictability of human behavior, the general belief that human behavior is not random, but rather complex, is also consistent with the notion of nonlinearity.

Chaos

The fact that nonlinear dynamical systems can exhibit complex and unpredictable behavior is interesting in and of itself and highly relevant for the study of human behavior. Even more interesting, however, is one of the key discoveries from the study of nonlinear dynamical systems: that highly complex behavior can even emerge from very simple rules or systems so long as the components or agents of the system interact in a nonlinear manner. That is, very simple deterministic nonlinear systems can produce extremely complex and unpredictable behavior. One notable form of complex behavior they produce is *chaotic behavior*.

A classic example of a simple nonlinear system that results in chaotic behavior is the *logistic map*. The logistic map is a discrete dynamical system, meaning its behavior changes over discrete rather than continuous time steps (see next section for more details). More precisely, it is simple dynamical equation of the form

$$x_{(t+1)} = rx_{(t)}(1 - x_{(t)}) \tag{11.1}$$

where x is the behavioral variable, r is a fixed behavioral parameter, and t equals time from step 0 to step n (i.e., $t = 0, 1, 2 \ldots, n$). To help make this equation less abstract and easier to understand, let us assume that this equation is used to model the daily mood of an individual diagnosed with bipolar disorder or manic depression, where x represents the individual's daily mood on a scale of 0 to 1, with 1 corresponding to a perfectly positive mood, and r representing the severity of the individual's diagnosis on a scale of 0 to 4, with 4 corresponding to a severe diagnosis. The predicted daily mood of the individual (i.e., $x_{(t+1)}$) is therefore a simple mathematical function of the current day's mood, $x_{(t)}$, multiplied by the severity of the diagnosis, r, multiplied by 1 minus the current day's mood, *or* $1-x_{(t)}$. For illustrative purposes, imagine that an individual's diagnosis was relatively severe, that r equaled 3.8, and the person's initial mood of x on day zero equaled 0.6. If we then computed or iterated the equation 100 times, we would get the time-evolving behavioral pattern displayed in Figure 11.1, where the value of x, or mood in our example, is plotted as a function of time step from 1 to 100. The complex and unpredictable nature of the behavioral pattern over time is quite evident, with the value of x from one time step to the next seeming to change in a way that far exceeds the simplicity of the equation that was used to generate it.

It is the latter feature of chaotic systems like the logistic map (Equation 11.1) that often surprises researchers. That is, while the behavior of such systems is completely determined by a set of simple deterministic (i.e., nonrandom) equations or rules, can be very complex and very difficult to predict. This is because chaotic systems exhibit a *sensitive dependence on initial conditions*: minor differences in starting states can become amplified as the system evolves over time. Given that measuring or knowing the initial conditions of any natural dynamical system with perfect precision is impossible, it is therefore equally impossible to predict the long-term behavior of such systems if they are chaotic.

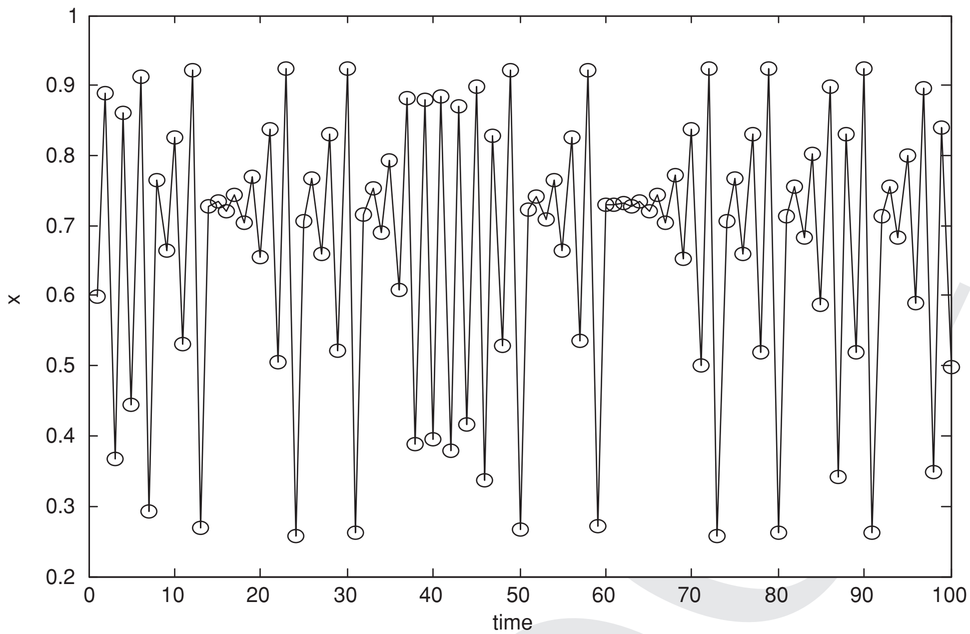


Figure 11.1. Behavior of the logistic map's dependent variable, *x*, during a chaotic regime (i.e., $r = 3.8$).

The implication of this principle for understanding human social behavior is quite profound. Most obvious is that the existence of chaos forces researchers to truly reconsider what it means for a behavioral event to be random (Guastello & Liebovitch, 2009). Perhaps less obvious is that the apparent randomness of variable or noisy behavior (i.e., normally distributed random noise or variance) that is traditionally assumed in nearly all psychology studies becomes an empirical question. Just because a variable behavior appears to be random does not mean that one should conclude that it is random. Chaos also highlights the nonobvious connection between past and future events, and that even extremely trivial changes can have a significant effect on the outcomes of time-evolving behavior. It is for these reasons that chaos has found its way into extensive theoretical discussion of social psychological systems. A lucid introduction and review of this point is found in Barton (1994). One prominent application described in Barton's review is in the clinical realm, where chaos has enjoyed a rapid growth of application across a range of traditions. Another popular, visual introduction to chaos and nonlinear dynamics that utilizes clinical psychology is provided by Abraham, Abraham, and Shaw (1990).

DYNAMICAL SYSTEMS MODELING

The goal of many research endeavors is to effectively model a behavioral system in order to make specific predictions about how the system will behave in the future. In personality and social psychology, this has typically been done using various forms of regression analysis and structural equation modeling (for instance, see Fabrigar and Wegener, Chapter 19 in this volume). An advantage of dynamical models is that they can handle the time-dependence of behavior and are not restricted to making linear assumptions about behavioral organization. Accordingly, nonlinear dynamical models can provide deep insights about the behavior of real-world time-evolving processes and can play a significant role in theorizing about how and why certain behavioral processes might emerge. For example, Meadows (2008) offers a lucid discussion of the benefits of dynamical systems modeling for understanding the interactions and behavioral nonlinearities that anchor social systems, from the parameters that lead to stable sustenance of romantic relationships (Gottman, Murray, Swanson, Tyson, & Swanson, 2003) to the way that societal rules can change patterns of self-organization.

It is important to keep in mind that any model of a biological, psychological or social system is at best an idealization (this is just as true for dynamical models as for non-dynamical models) and that the goal of a dynamical model is to capture the most important features of a system or process (Bertuglia & Vaio, 2005). Dynamical modeling involves describing how the behavioral state of a system changes over time. Here the term "state" refers to the current value of a variable (or variables) that are used to capture the system in question. A state variable could be any property of a system that might change over *time*, such as the movements of the body, limb, or eyes during social interaction, or the mood, attitudes, or personality characteristics of a child or adult. One could even model the change in state of *two* people, such as the quality of a romantic relationship. Essentially, a dynamical model describes how the state variables of a system evolve over time through rules or equations that determine the system's future state given its current state.

Difference Equations

A difference equation is a *recursive function* sometimes called an *iterative map* and can be used to model the behavior of a system at discrete time steps (1, 2, 3 . . . *t*, where *t* equals the number of time steps), with the state of the system at each time step defined as a function of the preceding state. The logistic map, introduced in Equation 11.1 in the previous section, is an example of a difference equation. More precisely, the logistic map is a one-dimensional nonlinear[1] difference equation – the dimension of a dynamical system equals the number of state variables needed to completely describe the system (i.e., one-, two-, three-, . . . to *n*-dimensions) – and is one of the most well-known and studied difference equations in the field of nonlinear dynamics. This is because despite the simplicity of Equation 11.1, it exhibits a wide variety of dynamic behaviors for different values of the system parameter r (specifically for $0 < r < 4$). This includes various types of stable fixed point and periodic behaviors, and, as we have already seen, even chaotic behavior (see Figure 11.2).

Of more relevance to understanding social behavior, Nowak and Vallacher have demonstrated how two coupled logistic equations can be used to model the behavioral synchronization of two individuals in social interaction (e.g., Nowak & Vallacher, 1998; Nowak, Vallacher, & Borkowski, 2002; Vallacher, Nowak & Zochowski, 2005; also see Buder, 1991). Their model takes the form,

$$x_{1(t+1)} = \frac{r_1 x_{1(t)}(1 - x_{1(t)}) + \alpha r_2 x_{2(t)}\ (1 - x_{2(t)})}{1 + \alpha} \qquad (11.2)$$

$$x_{2(t+1)} = \frac{r_2 x_{2(t)}(1 - x_{2(t)}) + \alpha_1 r_1 x_{1(t)}(1 - x_{1(t)})}{1 + \alpha}$$

where x is a generic variable representing the intensity of some observable, communicative behavior, such as approach (or avoidance), and r corresponds to internal states, such as personality traits, moods, attitudes, and values, that shape an individual's behavior over time. This is a two-dimensional model, in that there are two state variables, x_1 and x_2, one state variable and equation for each individual's behavior. The equations are also *coupled*, in that the behavioral state of each individual is dependent on his or her own previous state, as well as the previous state of his or her partner, with the parameter α (alpha) determining the strength of the coupling (i.e., mutual influence). Although a detailed discussion of this model and the behaviors it generates is beyond the scope of the chapter, its significance is that it predicts increased social monitoring and mutual influence (i.e., communication, mutual reinforcement and self- and other-monitoring) when individuals with different internal states (e.g., personality traits, moods, attitudes) are required to synchronize their behavior. This increase in social monitoring and mutual influence is then presumed to put greater stress on the interactional system by reducing the executive resources. In contrast, when partners have similar internal states, behavioral synchronization can occur with little social monitoring or mutual influence, leaving more energy and cognitive resources available for the coupled individuals to pursue common goals.

Differential Equations

In contrast to difference equations, which model the behavior of state variables across discrete time steps, a *differential equation* is a mathematical equation that models the *continuous time evolution* of a system in terms of the *rate of change* of state variables over time. As a starting example, consider the simple one-dimensional nonlinear differential equation

$$\dot{x} = rx(1 - x) \qquad (11.3)$$

where x is the state variable, $\dot{x}$ is the rate of change of the x over time, and r is a state parameter. This equation is very similar to Equation 11.2 and is the logistic equation in differential form. However, because Equation 11.3 models the rate and direction in which x

[1] It is a nonlinear equation because if we expand $rx_{(t)}(1 - x_{(t)})$, we get $r(x_{(t)} - x^2_{(t)})$, such that the state of the system is the product of a constant and the state variable to a power greater than one (e.g., $x^2, x^3, x^4 \ldots$), in this case $x^2_{(t)}$.

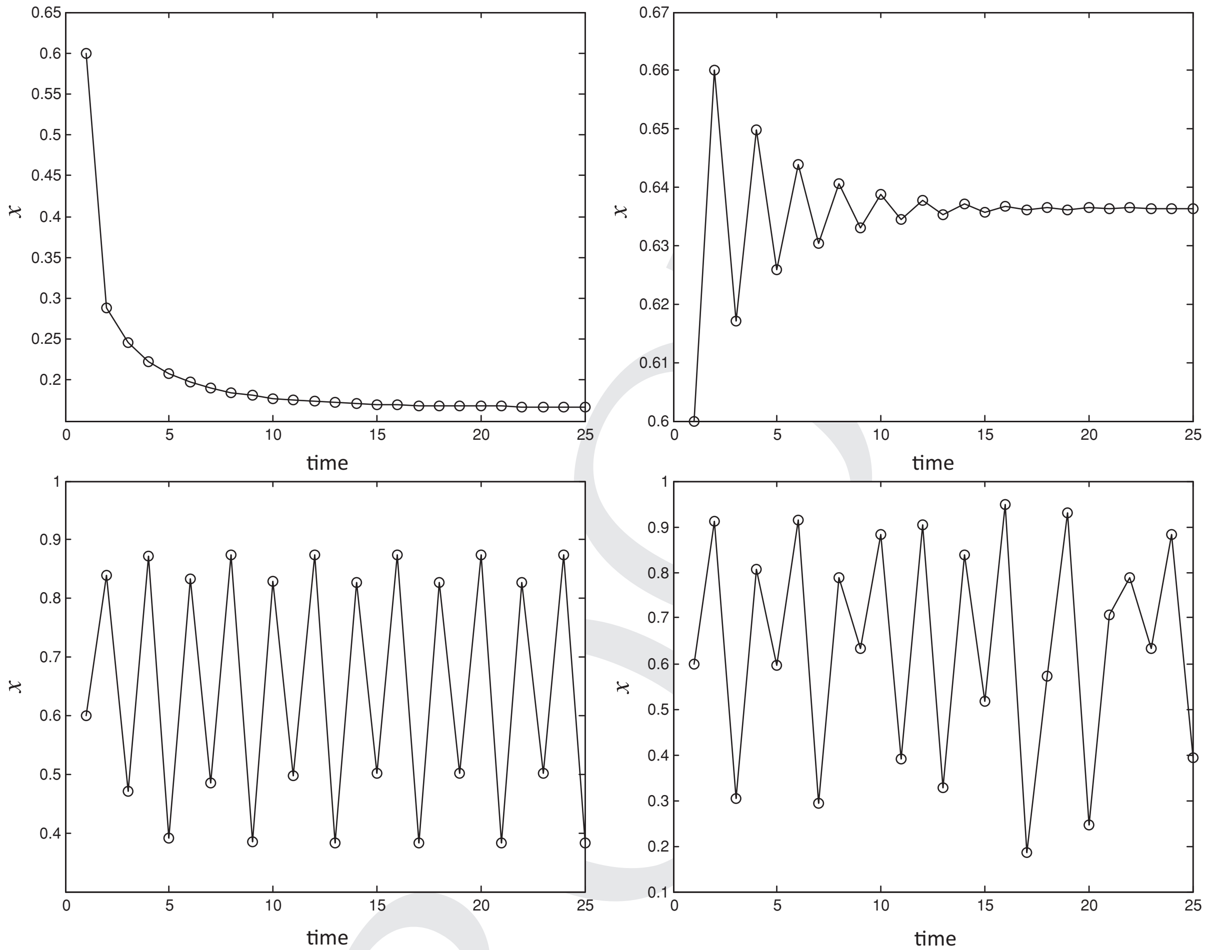


Figure 11.2. Examples of how x changes over time for the logistic map (Equation 11.1) for different values of r, but the same initial condition of $x_{(0)} = .6$. (top left) monotonic approach towards a fixed value of x for $r = 1.2$. (top right) oscillatory approach towards a fixed value of x for $r = 2.7$. (bottom left) periodical oscillates between 4 values of x for $r = 3.2$. (bottom right) chaotic behavior for $r = 3.8$.

changes over continuous time, the way the value of x changes is quite different from that determined by Equation 11.2. Imagine that x represents some observable behavior, such as *attraction* – in the context of social interaction, attraction could refer to the pull toward another person, affiliation, or liking – and r corresponds to the number of positive attributes. If we restrict our consideration to initial conditions, $x_{(0)} > 0$, and $r > 0$, then x always approaches $x = 1$, no matter what initial condition we choose. In other words, setting the parameter $r > 0$ would always predict the same eventual level of attraction. This can be seen from an inspection of Figure 11.3, in which the change in x over time is presented for four different initial conditions ($x_{(0)} = .1, .8, 1.6, 3.9$). Notice also that increasing r only changes the rate at which x approaches 1. It does not change the dynamics qualitatively, as it did for the logistic map in Equation 11.1. Because x is always attracted toward 1, irrespective of initial condition, $x = 1$ is the stable fixed point for Equation 11.3. We describe stable *fixed point attractors*, as well as other types of attractors, in more detail later in the chapter. At this stage it is sufficient to say that a stable fixed point is a state toward which the variables of a dynamical system move over time.

Although the simplicity of Equation 11.3 provides a good introduction to differential equations, the fact that its state variable, x, is always attracted toward 1 means that the degree to which this equation could be used to model complex human and social behavior is

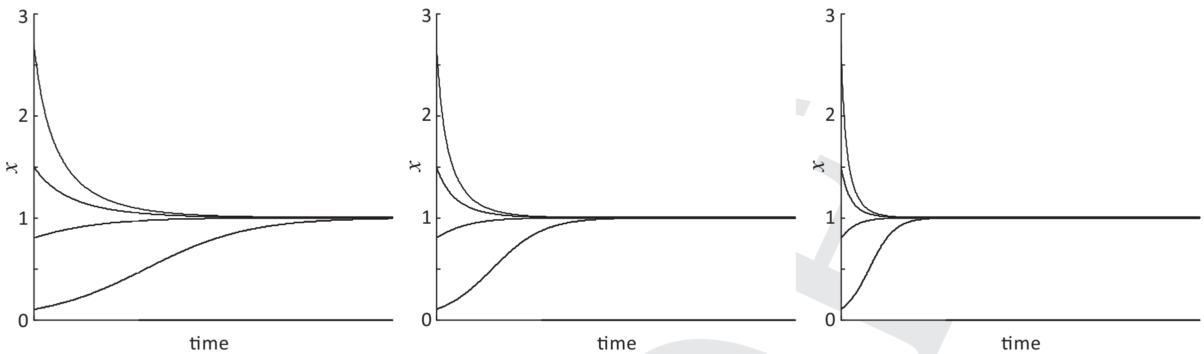


Figure 11.3. Examples of how x changes over time for Equation 11.3, using four different initial conditions ($x_{(0)} = .1$, $.8, 1.6, 3.9$). For the left, middle, and right graphs, $r = .5$, 1, and 2, respectively.

extremely limited. An example of a differential equation that is more relevant to understanding human and social behavior is

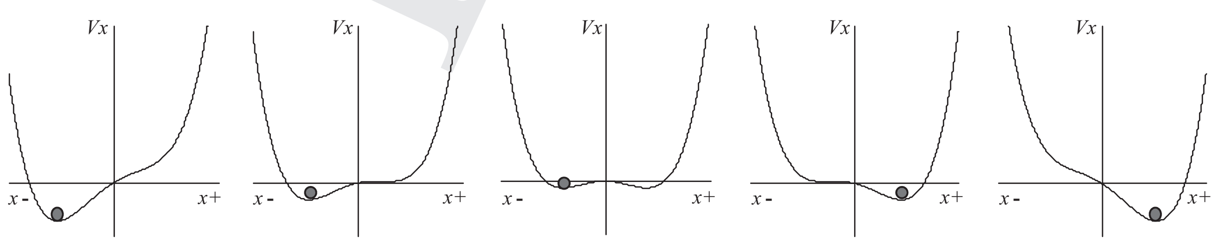
$$\dot{x} = k + x - x^3 \tag{11.4}$$

where x might represent an individual's attitude toward a certain political or racial group X and k is a state parameter that captures the amount of positive or negative experiences or information an individual has about group X. What is interesting about this system is that the number and location of its stable fixed points change as we change the parameter k. This is best illustrated by plotting Equation 11.4 as a potential function where the fixed points of the system are represented as wells in a one-dimensional landscape, with the depth of the well corresponding to the strength or stability of the stable fixed points or system attractors (see Figure 11.4).

Of particular relevance is that Equation 11.4 predicts that an individual's attitude toward group X would remain relatively stable, either negative or positive, across a range of k values and then at certain values of k suddenly transition from a negative to a positive state or from a positive to a negative state. More specifically, if an individual starts out with a negative attitude toward group X, and then k is increased from –1 to 1 (i.e., an individual starts to have more and more positive experiences with group X or receive more and more positive information about group X), the individual's attitude will be predicted to remain negative even beyond the point (i.e., $k > 0$) at which the individual has more positive than negative experiences with group X. The individual will finally transition to having a positive attitude toward group X after a critical number of positive experiences have occurred, that is, at $k = .35$. Conversely, if k is decreased from 1 to –1, a transition from a positive to a negative attitude will occur at $k = -.35$.

The kind of sudden transition predicted by Equation 11.4 is called a phase transition, where *phase* refers to a qualitative state of the system (i.e., a negative state or phase vs. a positive state or phase). Such

Figure 11.4. Potential functions for Equation 11.4, demonstrating how the system's stable fixed points change as k is scaled from –1 to 1 (left to right, respectively). In these plots, the x-axis corresponds to the possible values of the state variable, x, and the y-axis corresponds to the potential (Vx) of the system state to move to another state. Wells in the potential function or local minima (minimal potentials) correspond to stable fixed points, such that the state of the system (i.e., illustrated as a small grey ball) is trapped at the bottom of the well until that state is no longer stable (no longer a minima).



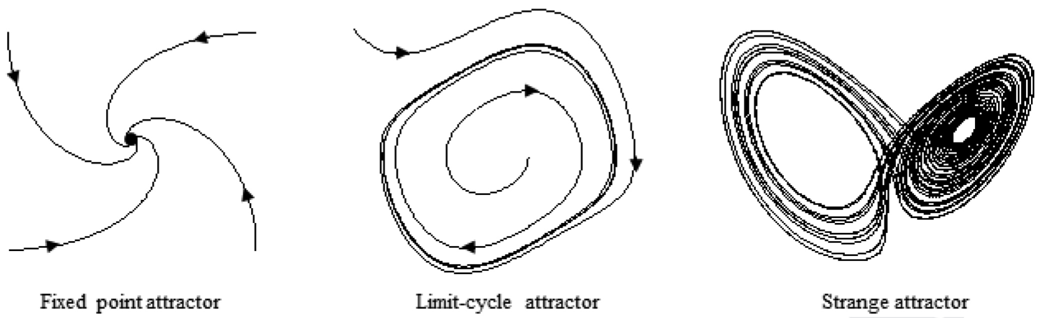


Figure 11.5. Prototypical examples of (left) a stable fixed point attractor, (middle) a limit cycle attractor, and (right) a strange attractor (see text for details).

transitions are a prominent characteristic of complex dynamical systems, and the fact that nonlinear differential equation models can be used to capture such behavior is part of their appeal. As the attitude example suggests, systems like Equation 11.4 are well suited for modeling qualitative changes in human cognition and social behavior. For instance, similar nonlinear systems can be employed to model transitions in categorical speech perception (Tuller, 2004; Tuller et al., 1994) and between conciliation and aggression during conflict situations (Colman, Vallacher, Nowak, & Bui-Wrzosinska, 2007).

Equations 11.2 and 11.3 are examples of one-dimensional differential equations because they have one state variable. Like difference equations, differential systems might also require more than one state variable to describe the behavior of the system. For instance, the motion of a pendulum requires a two-dimensional or second-order model that determines both its position and velocity over time. Differential systems with two or more state variables can also be coupled. For example, a two-dimensional system of coupled differential equations could be used to represent the population of two codependent species of animal (i.e., rabbits and foxes). With respect to personality and social psychology, two-dimensional systems have been used to model the coordinated behavior of interacting individuals (e.g., Baron et al., 1994; Schmidt &; Richardson, 2008; Tesser & Achee, 1994) and the long-term marital success of a husband and wife (Gottman et al., 2002; Gottman et al., 2003).

Attractors

One of the key aims of dynamical modeling is to effectively capture the attractors of a system. An attractor is a state or subset of states toward which the dynamical system moves over time (i.e., corresponds to a final future state or set of states). Attractors are often described geometrically, such as a point or a closed curve, and can be intuitively visualized by plotting the time-evolving behavior of a dynamical system in its phase space. A *phase space* is the set of all possible states of a dynamical system, with each state corresponding to a unique point in phase space. A graphical depiction of a system's attractors and the set of possible trajectories that can be exhibited by a system given its attractor layout is called a *phase portrait* (see Figure 11.5).

The attractor concept has been fruitfully applied in many social contexts. For example, Gottman et al. (2002) have developed models in which the stabilities of intimate relationships can be captured using fixed point attractors. As defined earlier, a *fixed point attractor*, commonly referred to as a *stable fixed point* (sometimes called a stable node, equilibrium point, or point attractor), is a location in phase space that system trajectories converge toward as time increases (Figure 11.5 left). A system may have two or more stable fixed points, in which case the system is considered to be *multistable*. The models developed by Gottman and colleagues often have two fixed point attractors: sustained marriage or divorce. The authors design differential equations much like Equation 11.4 and explore the parameters that govern the dynamic regimes of a married couple into one of these two equilibrium states. Although linear models can be applied in marriage research, the goal of the Gottman and colleagues' research agenda is to articulate the dynamic change – the trajectory that a couple may take – into one or another stable attractor (divorce, marital misery, happiness, etc.).

Space limits our discussion of all of the different types of attractors, but it is important to note two other types of attractor often featured in dynamical systems research, namely limit cycle and strange attractors. A *limit cycle attractor* is a subset of states within a system's phase space that make up a closed orbit that system trajectories converge toward as time increases (see Figure 11.5, middle). Limit cycles are paths that the system revisits with great regularity – such that, after a time, the system's behavior is fixed along the path (it is limited within that cycle). This kind of attractor is characteristic of periodic behaviors that exhibit the same stable spatial-temporal pattern over time (e.g., cycle with a stable frequency and amplitude over time) and, moreover, return to the same stable spatial-temporal pattern when perturbed. Rhythmic body and gestural movements provide classic examples and researchers have modeled interlimb (Haken, Kelso & Bunz, 1985) and interpersonal movement dynamics (Schmidt, Carello, & Turvey, 1990), and even complex social behavior, such as speech turns (Buder, 1991), using limit cycle models.

A *strange attractor* is a complex subset of states within a systems phase space that the state variable(s) of a dynamical system evolve toward over time. The term "complex" refers to the fact strange attractors have a non-integer or *fractal dimension*. That is, the spatial dimension of the subset of states that make up the attractor does not equal the standard Euclidean integer dimensions of one, two or three (e.g., a fractal attractor might live in a two- or three-dimensional state space, but is neither two-dimensional nor three-dimensional, but something in between). Strange attractors are characteristic of chaotic systems (although not exclusively) and in such cases are sometimes referred to as chaotic attractors. The strange attractor displayed on the right of Figure 11.5 is the chaotic attractor of the Lorenz system. The Lorenz attractor is made up of a complex subset of states within a three-dimensional space and has a fractal dimension of approximately 2.05. The set of differential equations that make up the Lorenz system was originally formulated by Edward Lorenz to model atmospheric convection, but it is the strange, butterfly-like structure of its chaotic attractor that has made it so famous.

Order and Control Parameters

Most of the models we have discussed so far have involved state variables that represent an individual behavioral quantity or element. For many complex systems, however, it would be impossible or unfeasible to have a different state variable and equation for every element, agent, or behavioral quantity entailed by the system. For example, if one were trying to model the behavior of gas molecules within a sealed container, it would be impossible to have a set of equations specifying the position and velocity of every molecule. Likewise, if one were attempting to model the movement synchronization that occurs between the members of an audience at a rock concert, it would be impractical to describe the position and velocity of each individual's postural and gestural movements over time. In these and other much simpler cases, it is often better to devise a dynamical model that effectively describes the macroscopic behavior of the system as a whole. This involves identifying a state variable that is able to capture the global or collective organization of the system. Such variables are referred to as *collective variables* or *order parameters.*

Identifying an appropriate order parameter is not always easy and often requires a significant amount of theoretical and empirical work, as well as model testing and assessment (for more details see Kelso, 1995; Nowak & Vallacher, 1998). Once an appropriate order parameter is identified, however, modeling and understanding the dynamics of a complex system is typically much simpler. For example, in the work of Vallacher and colleagues reviewed earlier, where the behavioral synchronization of two individuals is modeled using coupled logistic equations (Equation 11.2), the researchers chose a generic variable they called observable-communicative behavior, and conceived of this as a general patterning of behavior of one person relative to another in interaction. This order parameter is simply the intensity of overall behavior during interaction, which the authors suppose can be modeled as the magnitude of a single dynamical system's state variable. In reality, the social agents in this model are surely employing a whole range of observable behaviors, but by distilling these behaviors into one proposed order parameter, the model is not only more tractable and understandable but also more generalizable.

Another common example of an order parameter is *relative phase,* which captures the location of one periodic or rhythmic behavior in its cycle relative to another. For example, two people walking down the street have the swaying of their arms and legs in a relationship of relative phase. Relative phase is an order parameter because it quantifies in a single measure the spatial-temporal relationship between two periodic or rhythmic behaviors using a single variable. First employed to capture the stable patterns of synchrony that occur between two mechanical oscillators (e.g., coupled pendulum clocks), it has since been

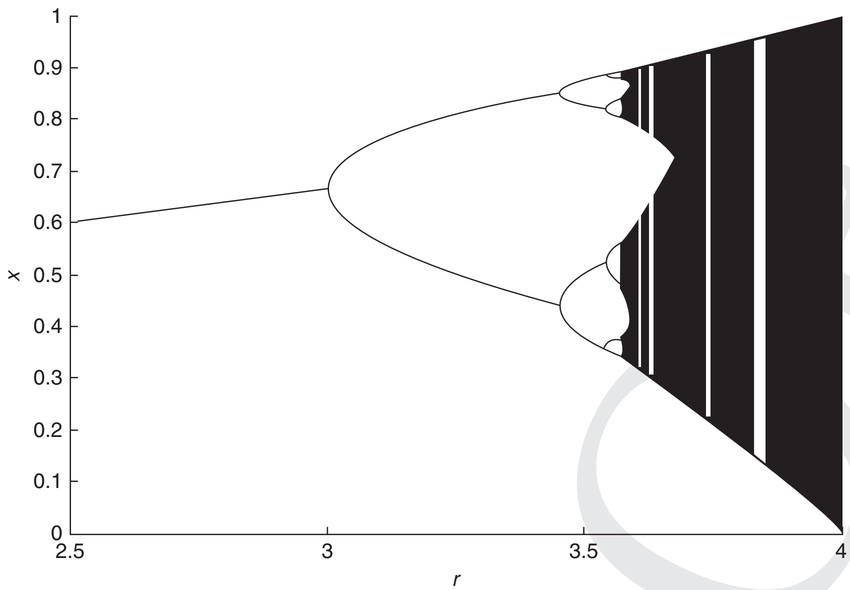


Figure 11.6. Bifurcation diagram for the logistic map, Equation 11.1.

employed to describe numerous biological phenomena, including the behavioral synchrony and social coordination that occurs between the bodily movements of two interacting individuals (see Schmidt & Richardson, 2008 for a review).

A *control parameter* is a system parameter that, when changed, can significantly influence the dynamical regime exhibited by a system. These parameters typically represent some external force, condition, or factor that plays an important role in constraining how a system can evolve over time, and sometimes even the attractors of a system. This is in contrast to other system parameters that are typically fixed and represent non-changing constraints or forces. In the discussion of how an individual's attitude toward group X might be modeled using Equation 11.4, parameter k operated as a control parameter. k represented the amount of positive or negative experiences or information an individual had about group X and increasing or decreasing k influenced attitude strength, with the attitude of individuals toward group X eventually transitioning from negative to positive or vice versa as k was scaled past some critical value.

Bifurcations

Bifurcations are changes in the number and/or type of fixed points or attractors that constrain a system's time evolution and take place when a system's control parameter reaches a particular critical value. Many of the systems described earlier exhibit bifurcations. For instance, the logistic map (Equation 11.2) exhibits a series of bifurcations as the parameter r is increased from 0 to 4. The differential system defined by Equation 11.4 has two bifurcation points, one when $k = +.35$ and one at $k = -.35$. In fact, nearly every system described previously exhibits one or more bifurcations as the values of certain control parameters are increased or decreased.

The possible attractors of a dynamical system that emerge or are destroyed as a control parameter is scaled can be visualized using a bifurcation diagram. The bifurcation diagram for the logistic map (Equation 11.2) as a function of the parameter r scaled from 2.5 to 4 is displayed in Figure 11.6. The y-axis corresponds to the possible long-term values of $x_{(t)}$, with r plotted along the x-axis. It is easy to see from this diagram that the logistic map has a single fixed point for $r < 3$, with the long-term behavior of $x_{(t)}$ equaling a single value of x. At $r = 3$, however, a bifurcation occurs with the possible long-term behavior equaling two fixed point values of x. The number of fixed points then continues to fork or double as r is further increased (e.g., from 2 to 4, to 8 to 16 . . . fixed points) and eventually exhibits chaotic behavior, with the possible long-term behavior of $x_{(t)}$ equaling all possible values of x.

There are numerous types of bifurcations and not all of them need to be described here (for more information about the different types of bifurcations,

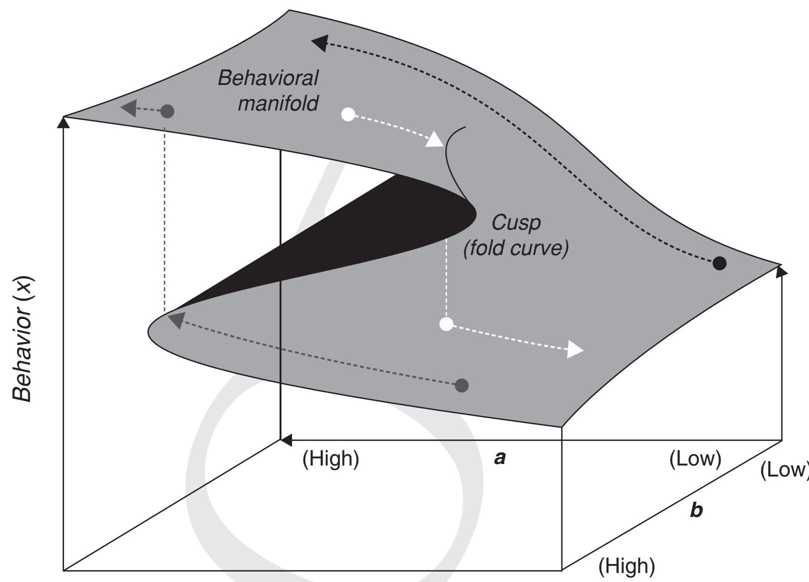


Figure 11.7. Behavioral manifold for the cusp catastrophe. Each point on the grey surface correspond to the stable fixed points (behavioral attractors) for different a and b values. The dashed lines illustrate to the different types of behavioral transitions that are possible when b is fixed at some value and a is scaled up or down.

see Strogatz, 1994). One theoretical class of bifurcations that are important to mention here are known as *catastrophes*. They are called so because they reflect a qualitatively dramatic change in the behavior of a system. What is particularly interesting about catastrophes and the models used to describe them is that they have proven to be theoretically useful for understanding the sudden emergence and or destruction of a range of social phenomena, including attitudes, self-evaluation, conformity, social relationships, and other catastrophic social transitions (e.g., Guastello, 1995; Latane & Nowak, 1994; van der Mass, Kolstein, & van der Pligt, 2003; Vallacher & Nowak, 1998).

Many of the catastrophic social transitions just mentioned have been modeled using the *cusp catastrophe*. The cusp model is a two-parameter version of Equation 11.4, namely

$$\dot{x} = a + bx - x^3 \tag{11.5}$$

where x is the state variable and a and b are systems parameters. The benefit of this model over Equation 11.4 is that it can be used to model how the interaction of two different external forces can influence behavior. For instance, this model can be used to describe the sudden changes in the dating behavior of couples given two interacting forces: a = love and b = social pressure (Tesser, 1980; Tesser & Achee, 1994). Here, social pressure refers to any family or societal pressure on an individual "not to date" a certain type of individual or group of individuals. For instance, an individual who has a conservative upbringing and whose friends and family are socially and politically conservative might be pressured not to date an individual who has had a very liberal upbringing and whose friends and family are all very liberal. Thus, social pressure is known as a splitting factor or parameter in Equation 11.5.

The bifurcation diagram for Equation 11.5 is displayed in Figure 11.7. The diagram is three-dimensional, with dating behavior, x, on the vertical axis and the control parameters for love, a, and social pressure, b, defining a control surface on the horizontal plane. The folded manifold plotted across the three axis is a manifold of the fixed points that exist for different settings of a and b. That is, each point on the manifold represents a fixed point, with the points on the non-folded (light grey) area of the manifold corresponding to stable fixed points and the points on the folded (dark) area corresponding to unstable (repulsive) fixed points. A close inspection of Figure 11.7 reveals when the model predicts that a catastrophic change in the dating behavior of a coupled would or would not occur. With respect to the latter case, in which social pressure, b, is low or zero, the prediction (and expectation) is that dating behavior will change almost linearly with changes in love. The more a couple loves each other – the higher the value of *a* – the more likely a couple is to date. If social pressure is high, however, increases or decreases in love have little effect on the likelihood that a couple will date, unless love increases or decreases past some critical value. At this point the couple will either suddenly enter into a strong dating relationship – if love is increased above a critical value – or suddenly stop dating altogether – if love is deceased below a critical value.

Cellular Automata, Agent-Based, and Artificial Neural Network Models

Modeling complex nonlinear dynamical systems using difference and differential equations is a powerful way of investigating the stability, dynamic patterns, and self-organization of systems. Using simple deterministic rules, with well-conceived variables and well-grounded parameters, can lead to a meaningful and generalizable theoretical understanding of the dynamics of various types of social behavior. There is, however, another set of methods for modeling the self-organizing dynamics of complex social systems, which starts from a microscopic level, focusing on how phenomena emerge from interacting elements or agents that behave according to simple rules. Such models are cellular automata and agent-based models.

A c*ellular automaton* is a collection of individual cells that form an *n*-dimensional grid of a specified size and shape, although they are typically restricted to either a one-dimensional line (array) or a two-dimensional square lattice of cells.[2] Although these cells could represent any component or element, in many social psychological applications a cell represents a person. The cells that make up a cellular automaton represent individual elements of the collective system (e.g., a community of individuals), with the behavior or state of each cell at any point in time determined by a set of simple rules that define its state based on the state of neighboring cells. In a two-dimensional cellular automaton, for example, a cell might be influenced by the four cells directly adjacent to its location (i.e., the cells above and below and to the left and right) or it might be influenced by all eight cells that surround it (i.e., the previous four, plus those on the diagonal).

Cellular automata, including one-dimensional automata, can exhibit a wide variety of complex behavioral patterns with emergent properties characteristic of complex dynamical systems (Wolfram, 2002). The rules that produce such patterns are usually deceptively simple given the complexity of consequences that can follow from them. Within social psychology, cellular automata have been used most notably to model the effects of social influence and public-opinion change based on Latané's social impact theory (for reviews, see Nowak & Lewenstein, 1996; Nowak & Vallacher, 1998). In these models, a two-dimensional cellular automaton is used with cells representing different individuals who possess an attitude of a certain strength toward a topic. The dynamics of an individual's attitudes are determined as a function of the attitude of near neighbors. After a number of iterations, this model eventually results in a stable pattern of attitudes across individuals (i.e., cells). Of particular interest is that the time-evolving patterns that result from this cellular automaton illustrate how key hypotheses from social impact theory play out, such as how pockets of minority opinion emerge over time.

Consider an example. In the models presented by Nowak and Lewenstein (1996), local interaction among cells of a cellular automaton can render small pockets or "walls" of resistance, where a cluster of cells is mutually supportive in sustaining their opinion, despite the onslaught of surrounding opinion. In additional simulations, they show that such minority opinion can grow to become the dominant one. For this to happen, the minority opinion is sparsely distributed in a social environment – weak and scattered. Yet, with a small bias to favor that minority opinion, things quickly change. What emerges in this model are "clusters" or "bubbles" of minority influence that, as they grow, come to interconnect with each other, thus growing in force and slowly dominating the once-majority opinion. Importantly, all of these simulations depend on simple, local interactions among cells, stretching across space and time.

An extension of classic cellular automata just described, which involve a collection of fixed cells whose state changes over time, is to have cells represent elemental locations, with the state of a cell corresponding to whether it is occupied or not. One of the earliest applications of cellular automata in the social sciences (Shelling, 1971) used this form of cellular automata and demonstrated how social segregation can result from individuals who are more dissimilar to those around them moving to a different location.

Agent-based modeling is another extension of cellular automata models. Such models have more potential

[2] A cellular automaton could also be a three-dimensional cube of cells, for instance.

for complexity, as agents in these models are not confined to a matrix of cells but are able to move around. One prominent use of agent-based modeling comes from the work of Axelrod (1984; Axelrod & Dion, 1988; Axelrod & Hamilton, 1981) on the emergence of cooperative behavior in games like the prisoner's dilemma. Axelrod's work demonstrates that strategies that are cooperative but punish defections (e.g., the tit-for-tat strategy) are most successful in computer simulation tournaments in the long run when pitted against agents using other strategies. Moreover, if the program increases the likelihood of an agent taking on the strategy of an interactant who is more successful in their game, the tit-for-tat strategy spreads in a population. Intriguingly for a dynamical perspective, history is crucial for learning such a strategy. If there is a reshuffling of agents after each trial so that they have no continuity in whom they interact with, then there is no means for an agent to learn personally the negative consequences of defecting when the other cooperates (punishment on the next trial) – the agents have by then moved on to the next partner. As a result, lacking history with an interactant cooperative strategies are not learned (Axelrod, Riolo, & Cohen, 2002) – the tit-for-tat strategy does not spread.

One interesting aspect of cellular automaton and agent-based models more broadly is the emergent phenomena that result from the structure of linkages between agents. Traditional social psychological methods do not account well for how being in contact with multiple individuals, repeatedly and over time, affects behavior (Mason, Conrey, & Smith, 2007). Because agent-based models do, they allow a researcher to see how novel phenomena may emerge as a consequence of the interdependencies among agents (Smith & Conrey, 2007). A recent agent-based simulation of impression formation capitalizes on the unique potential for such models to examine the consequences of flow of information across different types of linkages (Smith & Collins, 2009). In Smith and Collins's model, participants can sample information about another person directly or indirectly (e.g., through gossip). They used Kenny's Social Relations Model (1994) to analyze key features of impressions – for example, the degree to which impressions do in fact reflect commonalities (across perceivers) attributable to the target, versus how much variance is owing to perceiver and specific target-perceiver relationships. The results of their simulations indicate that the ways of obtaining information matter. The most negative impression came from one-sided elicitation (because people are likely to cease seeking information if their initial impression is negative). Sampling information socially led to more positive impressions on average, as well as reduced perceiver effects and relationship effects. Moreover, the authors found emergent phenomena that had to do with dyadic reciprocity and the relative accuracy of generalized versus dyadic accuracy (Smith & Collins, 2009).

In the Smith and Collins's (2009) model, although the model is directed toward understanding cognitive processes through transmission of information about a person (directly or third-person), the interacting agents in the model are individuals. It is important to realize, however, that the agents in agent-based modeling can be the cognitive elements themselves (e.g., the interaction between visual perception, inferences, judgments, etc.). Thus, there is a close link between such models and dynamic models of memory. A brand of modeling that has focused entirely on exploration of the dynamics of social cognitive processes is *artificial neural network models* (also called connectionist models in the late 1990s). In a classic work in social psychology, Smith and DeCoster (1998) used a basic associationist neural network model to explore a wide range of phenomena, including stereotyping and person perception. They argued that neural network models of memory and social judgment – which function by integration of basic informational cues (much like interaction among agents) – can more parsimoniously account for a range of social phenomena than traditional models of memory (e.g., Wyer & Srull,1989). Modern neural network models can also be used to model cognitive dynamics, including the dynamics of social cognition; see Smith (1996) for an excellent early introduction to use of neural network (connectionist) models in social psychology.

Such models have also been applied to social interactive phenomena. Nowak and Vallacher (1998) review early models of interpersonal dynamics using neural network models. Perhaps the best illustration of such models is a very recent model meant to tap into the fast-time-scale dynamics of person perception and judgment by Freeman and Ambady (2011). They proposed that person construal (such as identifying the gender of a person) is constrained by a constellation of information sources in the environment, such as hair cues, skin cues, facial configural cues, and so on. They developed an interaction-activation framework (see collection of papers in Rumelhart & McClelland, 1985) in which cues are integrated *incrementally* and *probabilistically* in time. This provides an array of predictions about how personal construal *dynamics* is shaped and guided by different combinations of cues. The authors have explored such dynamics in a variety of experiments on the behavioral dynamics of these perceptions using, for example, mouse-tracking methods. In this behavioral approach, researchers can track participants as they move their computer mouse toward varying options on a screen (e.g., during person construal). These mouse movement trajectories can then be mapped directly onto a computational neural network model: The evolution of the computer cursor on the screen toward a response box can be captured by a neural network state, achieving some stable response activation over choices. Accordingly, the dynamical model, together with behavioral recordings, makes a tidy dynamical package for exploring the dynamics of social decision making, perception, and judgment. For a review of the behavioral results, see Freeman et al. (2011); for software that allows dynamic tracking of these social processes, see MouseTracker (Freeman & Ambady, 2010).

DYNAMICAL SYSTEMS ANALYSIS

Dynamical modeling is important because it provides researchers with a set of tools for understanding in an abstract and often extremely general way how human and social behavior can emerge. As many of the examples provided earlier highlight, the power of a dynamical model does not always rest on its ability to simulate real-world behavior, but rather whether it can generate testable predictions, enhance theoretical development, and motivate research questions. Unfortunately, there is no step-by-step guide that one can follow when developing dynamical models of human and social behavior. Building an effective model requires good understanding of the many different types of models and mathematical functions that can be employed to capture differing types of dynamics, as well as a significant amount of theorizing and lots of trial and error. A researcher interested in modeling the dynamics of a behavioral system also needs to have a good understanding of the system's underlying stabilities and its relevant state variables and parameters. In many instances, however, researchers in social-personality psychology do not start with sets of state variables, parameters, or mathematical functions or equations, and may not know the nature of a behavioral system's underlying dynamics. In such cases, research typically starts with a temporal sequence of behavioral measurements or observations – a *behavioral time series* – recorded during experimental, nonexperimental, or observational research. A researcher then attempts to uncover the dynamics of a behavior using various forms of time-series analysis. Accordingly, in this final section of the chapter we review some of the tools that can be employed for dynamical analysis of behavioral time series.

Before introducing various methods of dynamical time-series analysis, it is important to appreciate that empirical research and the analysis of behavioral time-series data can be, but is not always, a precursor to modeling. Rather, research and modeling are best conceptualized as complementary methods of dynamical analysis, with researchers often moving back and forth between both forms of research (i.e., behavioral research and dynamical modeling), using experimentation and time-series analysis to identify key state variables, attractors states, and control parameters, and mathematical modeling to better understand and test empirical findings and make future predictions. A detailed description of the nuances of how one goes from the dynamical analysis of behavioral time-series data to a dynamical model is well beyond the scope of this chapter. We do wish to emphasize, however, that building a dynamical model is not always necessary for understanding the dynamics of behavior. In many cases, building a model to simulate the dynamics uncovered via behavioral time-series analysis will not necessarily provide more insights or additional information about a system's underlying dynamics. Accordingly, many dynamical systems researchers are less concerned with building dynamical models and instead focus more of their efforts on uncovering the dynamics of behavioral systems via experimentation and the kinds of dynamical time-series analysis techniques outlined in the following sections.[3]

Behavioral Measurement

As with any research study, determining valid and reliable dependent variables is fundamental. What the right dependent variable is when investigating the dynamics of social phenomena will of course depend on what behavior is measurable in a given context, along with a researcher's theoretical interests.

[3] The converse is also true, with some researchers focusing primarily on building abstract dynamical models like those described in the previous section without collecting real behavioral data (time-series or otherwise). Just as valid as empirical research, this latter "research by modeling" approach is less concerned with simulating real-world behavior and is more concerned with developing a formal yet highly generalizable understanding of how organized behavior can emerge, change, and dissolve over time.

Obviously, the dependent variable should capture the *state* of the agent or system at the time of measurement. As we outlined previously, a behavioral state represents a wide array of measures. For example, socially relevant measures may come in the form of self-esteem, personality characteristics, attitude, or social dominance measured over time. A measured behavioral state could also be a mode or type of behavior, such as whether an individual carries an object alone or together with another person, or exclusionary acts of a pair of individuals with respect to some third (perhaps out-group) individual during an interactional game. In social-personality psychology, it is also common for researchers to record other more indirect or implicit measures of a behavioral state, including physiological measures such as heart rate, cortisol level (Dickerson & Kemeny, 2004), muscle activity, skin conductance, and neurophysiological measurement techniques (e.g., using EMG, EEG, or fMRI; see Berkman, Cunningham, & Lieberman, Chapter 7 in this volume; Cacioppo, Tassinary, & Berntson, 2007; Stam, 2005). Overt behaviors that have social relevance also include body position and movement (for reviews, see Fowler, Richardson, Marsh, & Shockley, 2008; Schmidt & Richardson, 2008) and eye gaze (Richardson & Dale, 2005). Any social process likely has a behavioral proxy that can be tracked, semi-continuously, in time.

No matter what dependent variable one chooses, there are several important requirements when investigating the dynamics of social behavior. First, and most obviously, the dependent variable must correspond to a behavioral state measurement that can be recorded repeatedly over time. This results in a sequence of measurements over time, or more specifically, a *behavioral time series*. Here, the term "time" could refer to "clock" time such as second, minute, hour, day, month, etc., or it could refer to some other time scale, such as trials, sessions, or events. That is, a researcher could record a behavior almost continuously, making measurements several times a second or after some longer time interval. For instance, a researcher could record an individual's postural position 50 times a second during the course of a 2-minute conversation (e.g., Schmidt, Fitzpatrick, Caron, & Mergeche, 2011; Shockley, Santana, & Folwer, 2003), the heart rates of an infant and mother sampled 1,000 times a second during a three-minute face-to-face interaction (Feldman, Magori-Cohen, Galili, Singer, & Louzoun, 2011), vocal activity assessed at fractions of a second (Warlaumont, Oller, Dale, Richards, Gilkerson, & Dongxin, 2010), respiration patterns assessed at each breath intake for dyads involved in lengthy casual conversations (McGarva & Warner, 2003; Warner, 1992; Warner, Waggener, & Kronauer, 1983), an individual's emotional expression twice a minute while watching an emotive film (e.g., Mauss, Levenson, McCarter, Wilhelm, & Gross, 2005), or an individual's self-esteem or mood every day over the course of two years (e.g., Delignières, Fortes, & Ninot, 2004).

Behavioral time series can also be sequences of discrete behavioral events (e.g., occurrence of categorical events or coded behaviors) that could be dichotomous (a single action occurs or not, such that the time series is a sequence of 1s and 0s) or that involve several different discrete states recorded on a nominal unit scale. For instance, a researcher could record which object is being looked at and in what sequence during a joint task (Richardson & Dale, 2005), which words and sequences of words are used by an individual or group of individuals during a conversation (Louwerse, Dale, Bard, & Jeuniaux, 2012), or a dichotomous coding of when individuals vocalize or not during interactions with someone they believe to be attitudinally similar or dissimilar to themselves (McGarva & Warner, 2003).

Irrespective of the type or time scale of the behavior being measured or recorded, the ordering of observations or measurements in the behavioral time series must be recorded sequentially in time. Extracting the emerging patterns or stable states of behavior from data requires historical information about the state of the system preceding its current state. If future observations depend on observations that preceded it in time – in other words, that are sequentially dependent – then data recorded nonsequentially will prevent identification of trends, stable states, or reoccurring patterns that may exist. In many dynamic analyses, an additional constraint is that the time intervals between sequential measurements must be the same. This is because the dynamic regimes that characterize many continuous behaviors have specific temporal properties that can only be determined if the time between measurements is known and equivalent.[4] For instance, many human and social behaviors are characterized by periodic patterns, from the intrinsic periodicity of brain-wave patterns (e.g., Tognoli, Lagarde, DeGuzman, & Kelso, 2007), to the leg and arm movements of an individual while walking (e.g., Moussaïd,

[4] Subtle variations in time interval are not always catastrophic, especially for longer intervals (i.e., hour or day), but should be minimized as much as possible.

Perozo, Garnier, Helbing, & Theraulaz, 2010), to the day-to-day mood of an entire population of people (Dodds, Harris, Kloumann, Bliss, & Danforth, 2011). In each case, the same or similar behavioral states recur again and again after a certain period of time (i.e., with a certain temporal frequency).

It is also crucial that the measurement device has enough resolution to reliably capture whether the behavioral state has changed across repeated measurements. In many instances, traditional methods of measurement, such as obtaining self-evaluations on a 7- or 9-point Likert scale, or simply coding whether or not a certain behavior or action occurred (i.e., 1 for yes and 0 for no), can provide the resolution needed. In other cases, such as when recording the subtle gestures, postures, or eye movements of conversing individuals, or the subtle changes in the body language, emotion, and anxiety of an individual during an interpersonal confrontation, more advanced methods of measurement might be required. Thankfully, recent technical progress has facilitated collection of such behavioral time series. For instance, modern video processing technology enables researchers to acquire objective whole-body activity time series of one or more individuals from synchronized multiview video recordings (e.g., Kupper et al, 2010; Schmidt, Morr, Fitzpatrick, & Richardson, 2012). There are also technologies for continuously recording an individual's movements in space and time, such as Polhemus tracking systems (e.g., Polhemus Liberty and Latus systems, Polhemus, Ltd, Virginaia) and NDI Optorack (Northern Digital Inc., Ontario, Canada). There are also now a number of low-cost off-the-shelf gaming systems (e.g., Nintendo Wii remotes and force plates, and the MS Kinect) that can be used to wirelessly record the movements of interacting individuals in real time. As for physiological measures, Biopac systems are now widely used for tracking one or more signals, and can be used as both an electroencephalogram and electromyogram (Biopac Systems, Inc., Aero Camino Goleta, CA). Blascovich (Chapter 6 in this volume) provides more details about physiological measurement.

With regard to recording more discrete behavior, even for eye movements and gestures, as well as language analysis, there are now hardware and software applications that can automatically categorize the behaviors being emitted. For language analysis, researchers relied on well-developed schemes in the psychological sciences for coding or transcribing social interaction, involving time-intensive practices that require careful selection of units of analysis, guided by research goals (Bakeman, Deckman, & Quera, 2005; Heyman, Lorber, Eddy, & West, Chapter 14 in this volume; Kreuz & Riordan, 2011). Such research can be facilitated by powerful annotation software (Loehr & Harper, 2003). Eye movement technologies can use "areas of interest" (AOI) to transform a sequence of x,y-coordinates on a computer screen to a set of looked-to objects. Researchers have also used continuously recorded body movements and speech to infer discretely labeled states. For example, in the domain of human-computer interaction, machine-learning algorithms have been applied to extract meaningful states (Castellano, Kessous, & Caridakis, 2008), such as which emotion is being experienced by a person, given multimodal cues such as speech, and face and body movements. In addition, gesture recognition algorithms allow hand gesture patterns (discrete categories) to be obtained by learning algorithms applied to body movement data (see Mitra & Acharya, 2007 for a review).

Methods of Dynamical Analysis

So what do you do after you have obtained a time-series recording of a behavioral phenomenon? How do you investigate the dynamics present in a recorded time series? In general, the dynamical analysis of a behavioral time series involves qualitative and graphical assessment of the time-evolving pattern of behavior and then quantitative linear and/or nonlinear time-series analysis.

Qualitative and graphical assessment. For any research study, knowing what the recorded data "look" like is essential for appropriate understanding and analysis. Although visual inspection alone does not typically reveal what the underlying dynamics are, it does provide a general understanding of the kinds of analysis techniques that will be needed to uncover the dynamics. For dynamical analysis, this first and foremost involves graphing the behavioral time series on a time-series plot and visually inspecting the patterns it contains. For illustrative purposes, hypothetical examples of some of the different kinds of time series that might be obtained in social-personality psychology are displayed in Figure 11.8 (consult sources cited later in this paragraph for data examples). An inspection of different time series highlights just a few of the many different types of behavioral time-series patterns that could be recorded. In some cases the patterns of change over time are relatively simple and regular: the monotonic decrease of an individual's anxiety level over the course of 50 therapy session (Heath, 2000) and the oscillatory movements of an individual's right

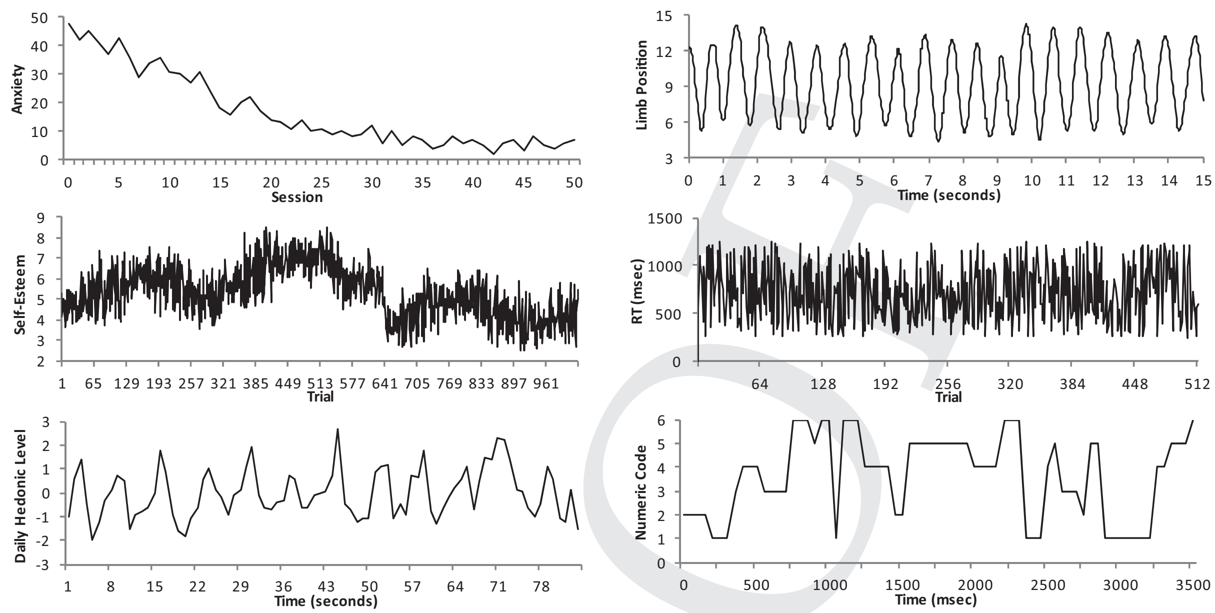


Figure 11.8. Hypothetical examples of several types of behavioral time series. (top left) Change in anxiety level for an individual over 50 therapy sessions. (middle left) An individual's self-esteem recorded on a 9-point Likert-scale twice a day for 512 days. (bottom left) An individual's daily hedonic (mood) level recorded over 12 weeks. (top right) Motion sensor recording of a individuals right arm movements while walking. (middle right) Reaction times of a participant completing a 512 trial lexical decision task. (bottom right) A time series representing categorical data obtained from eye movement behavior while a person views an array of 6 images. An eye-tracking system records a numeric identifier 1–6 reflecting which particular image is being fixated over time (see text for more details).

arm while walking (Harrison & Richardson, 2009). In other cases the patterns of change over time are highly complex and appear to be nondeterministic or stochastic (i.e., random): an individual's self-esteem over the course of 1.5 years (see Delignières et al., 2004) and the trial-by-trial RT and an individual completing a 512 trial lexical decision task (see Holden, 2005). Others seem to fall somewhere in between, containing semi-periodic patterns or other complex regularities. Two examples are the daily hedonic level or mood of individuals over the course of twelve weeks (see Larsen & Kasimatis, 1990) and the eye fixations that occur when an individual scans the world during a conversation (see Richardson & Dale, 2005).

In addition to inspecting time-series plots of one's data, plotting a behavioral time series as a trajectory in phase space can also provide researchers with a clear qualitative understanding of the attractor(s) that constrain the time-evolving behavior. If the state space of a behavioral systems is known a priori (and contains no more than three dimensions), this is often quite easy. It is more often the case, however, that the phase space of a behavioral system is not known a priori. In such cases, the process of plotting behavioral time series and a trajectory in phase space requires that one uncover – or more precisely, recover – the phase space of a behavioral system analytically. *Phase space reconstruction* involves a number of steps that enable a researcher to recover a phase space isomorphic to the system's real phase space. In short, phase space reconstruction involves extracting the entire multidimensional dynamics of a system, in all its complexity, from a one-dimensional time-series recording. Although a detailed description of the steps required to complete phase space reconstruction is too involved to be unpacked here (for a more detailed description and tutorial, see Abarbanel, 1996 and Kantz & Schreiber, 1997), the method itself is powerfully intuitive. For systems that have a phase space with more than three dimensions, phase space reconstruction also provides a quantifiable measure of a system's dimension – an indication of the number of the state variables required to model the system effectively. Examples of phase space reconstruction applied to a socially relevant phenomenon are described in Shockley et al. (2003) and Richardson, Schmidt and Kay (2007), who study behavioral synchrony between interacting partners. In this research, a single bodily movement measure – such as postural change – is recorded as a signature of the fluctuations of the total mind/body system. To gain access to the higher dimensions of the system from the (single) one-dimensional time-series measure (i.e., change in posture sway over time), researchers use phase space reconstruction to discern how many dimensions best capture the fluctuations observed in the movement time series (often as many as 10 dimensions). Once phase space is reconstructed, researchers can then mathematically compare how two people's movements change in relation to each other in this higher-dimensional space.

Linear and nonlinear time-series analysis. One of the key decisions a researcher must make when inspecting time-series plots is whether the dynamic regime that characterizes the behavior of interest is simple or regular enough to be analyzed using linear methods, or whether the behavioral dynamics are sufficiently complex that one must employ nonlinear methods. Unfortunately, there is no definitive rule as to when one should employ linear or nonlinear methods and in many instances, especially when performing a dynamical analysis on new phenomena or on behavioral time series that have not previously been examined, it is prudent to employ a range of linear and nonlinear methods in order to determine different aspects of the behavioral dynamics recorded.

In general terms, however, linear methods of analysis are preferable when the patterning of movement or behavior being investigated is highly regular and *stationary*. That is, the mean and dispersion of sampled values in the time series have a regular pattern and remain more or less the same across the interval of recorded time. The anxiety, daily hedonic level, and limb movement time series in Figure 11.8 all meet this criteria, as does the RT time series (although see section on fractal analysis later in the chapter). For time series data that is *nonstationary* – the mean and dispersion of sampled values vary markedly across the time-series recording – or for behavioral time-series that contain a high degree of stochastic variability or involve highly complex or aperiodic patterns of change over time, nonlinear methods may be more effective. What follows is a brief description of several common and generally applicable linear and nonlinear time-series methods for research in social-personality psychology (for more detailed discussion and tutorials, see Abarbanel, 1996; Boker & Wenger, 2007; Gottman, 1981; Heath, 2000; Kantz & Schreiber 1997; Riley & Van Orden, 2005).

Spectral analysis and cross-spectral coherence. One of the first questions commonly asked when analyzing time-series data is whether the data contains any periodic or temporal structure. Consider the limb movement and daily hedonic time series in Figure 11.8. Do the up and down fluctuations occur in a stable periodic manner? If so, after what period of time (i.e., at what frequency) does the pattern repeat itself? Conducting a spectral analysis enables one to answer these questions by decomposing a time series into its periodic components by estimating how well a set of sine or cosine functions of different frequencies and amplitudes fit the data. Performing a spectral analysis is much like conducting a regression analysis in that you are attempting to decompose the major sources of variation in the data, in this case trying to determine which component frequencies account for significant amounts of variability in the signal. For highly stable periodic behavior, like the rhythmic limb movements displayed in Figure 11.8, there is usually only one dominant or fundamental frequency component. For less stable periodic data, like the daily hedonic time series displayed in Figure 11.8, individual frequency components will be less powerful. There might also be more than one frequency component in a time series (i.e., multiple frequency components). This is particular true for highly complex or semi-periodic time series.

Spectral analysis can also be employed to determine how correlated two time series are by examining, essentially, the similarity of their frequency patterns. This comparison is called cross-spectral coherence and indexes the correlation between two time series on scale of 0 to 1, and is analogous to calculations of the squared correlation coefficient (Gottman, 1981; Porges et al., 1980; Warner, 1988). In social-personality research, cross-spectral coherence is commonly employed to examine mutual influence and behavioral coordination, that is, the degree to which one individual's behavior is influenced by and/or coordinated with the behavior of another. For example, Sadler et al. (2009) used cross-spectral methods to explore the rhythmic relationships between two people's dominance and affiliative dynamics during interaction. In this study, coders dynamically tracked interaction partners using a joystick, thus producing dominance/affiliation time series. The authors found that, indeed, pairs of interaction partners exhibit similar affiliation amplitude-frequency patterns. Put differently, they shared behavioral cycles.

Autocorrelation and cross-correlation. Dynamic human and social behavior is usually correlated over time. In other words, how an individual behaves at any given moment is typically correlated with how the individual behaved sometime recently. The dependence or correlation between future and past behavior can be determined using autocorrelation. Sometimes called *lagged correlation*, autocorrelation identifies if future states are correlated with past states by determining the correlation between points in a time series at different time lags. A positive autocorrelation indicates persistence of behavior after some time lag; the behavioral change is similar from one observation to the next (e.g., positive changes or state values in both past and future states). A negative autocorrelation indicates anti-persistence of behavior after some time lag; opposite behavioral change occurs from one observation to the next (e.g., positive changes or values in the past state correspond to negative changes or values in the future states).

Cross-correlation is a simple extension of autocorrelation and examines the dependence or correlation between future and past values of different time series. It is also commonly used to examine mutual influence and behavioral coordination, and yields similar results to coherence analysis (described earlier), except that one can also look at the correlation between individuals' behaviors at time lags other than zero. Accordingly, it can be employed to determine if two behaviors are attracted toward each other, and also whether one behavior leads or follows another behavior at some specific time lag.

Relative phase analysis. Another technique for examining mutual influence and behavioral coordination is relative phase analysis. This technique has been employed most extensively in research examining behavioral synchrony – the rhythmic movement coordination that occurs between the limb or body movements of interacting individuals (e.g., Marsh, Richardson, & Schmidt, 2009; Miles, Lumsden, Richardson, & Macrae, 2011; Schmidt & Richardson, 2008). It can also be employed to investigate the patterns of coordination that occur between any set of rhythmic or periodic behavior (e.g., the coordinated changes in day-to-day mood of husband and wife or mother and child). In short, the technique involves calculating the difference in the "phase" of two (or more) rhythmic or periodic behaviors over time (for details on multivariate relative phase analysis, see Frank and Richardson, 2010; Richardson, Garcia, Frank, Gregor, & Marsh, 2012). Here the term "phase" refers to the location of a system or behavior within its cycle. The relative phase or "difference in phase" between two rhythmic or periodic behaviors therefore corresponds to the location of one behavior within its cycle relative to the location of the other within its cycle. Thus, if the relative phase between two behavioral time series remains the same over time, the behavior is said to be coordinated at that relative phase relation. For example, consider two people coordinating their rhythmic gait while walking down the street together – their leg cycles are in the same place and are cycling together.

Typically, behavioral synchrony is constrained to two stable patterns of behavioral coordination over time, commonly referred to as *inphase* and *antiphase* coordination (Haken, Kelso & Bunz, 1985; Schmidt, Carello, & Turvey, 1990). Inphase coordination corresponds to rhythmic or periodic movements or behaviors that move or change in the same direction at the same time (such as the walkers we just mentioned). Antiphase coordination corresponds to rhythmic or periodic movements or behaviors that move or change in the opposite direction at the same time. To use the walking example again, this would mean that individuals would be continuously moving their legs in opposite patterns – as one person swings her right leg forward, the other would be swinging her right leg back. It is worth noting that these latter descriptions of inphase and antiphase coordination characterize perfect or absolute synchrony. During natural social interaction, however, the movements or behavior individuals do not usually become coordinated in a perfect inphase or antiphase manner, but rather exhibit intermittent periods of inphase and/or antiphase coordination (e.g., Richardson, Schmidt, & Kay, 2007; Schmidt & O'Brien, 1997).

Recurrence analysis. The analysis methods discussed so far are based primarily on assumptions of linear relations that underlie most analyses commonly used in psychology (e.g., ANOVA). Accordingly, they are only able to capture the linear dynamics of stationary time-series data. Recurrence analysis, however, is a nonlinear analysis method and can be employed to analyze both stationary and nonstationary data. Indeed, the beauty of recurrence analysis, in comparison to other linear time-series methods, is that it does not require assumptions about the structure of the time series being investigated or the underlying dynamics that shape the recorded structure: The behavior can be periodic, nonperiodic, or stochastic, even discrete or categorical.

Although recurrence analysis is still relatively new, particularly in psychology (e.g., Riley, Balasubramaniam, & Turvey, 1999; Shockley, Santana, & Fowler, 2003), there is now substantial evidence that suggests it is potentially one of the most robust and generally

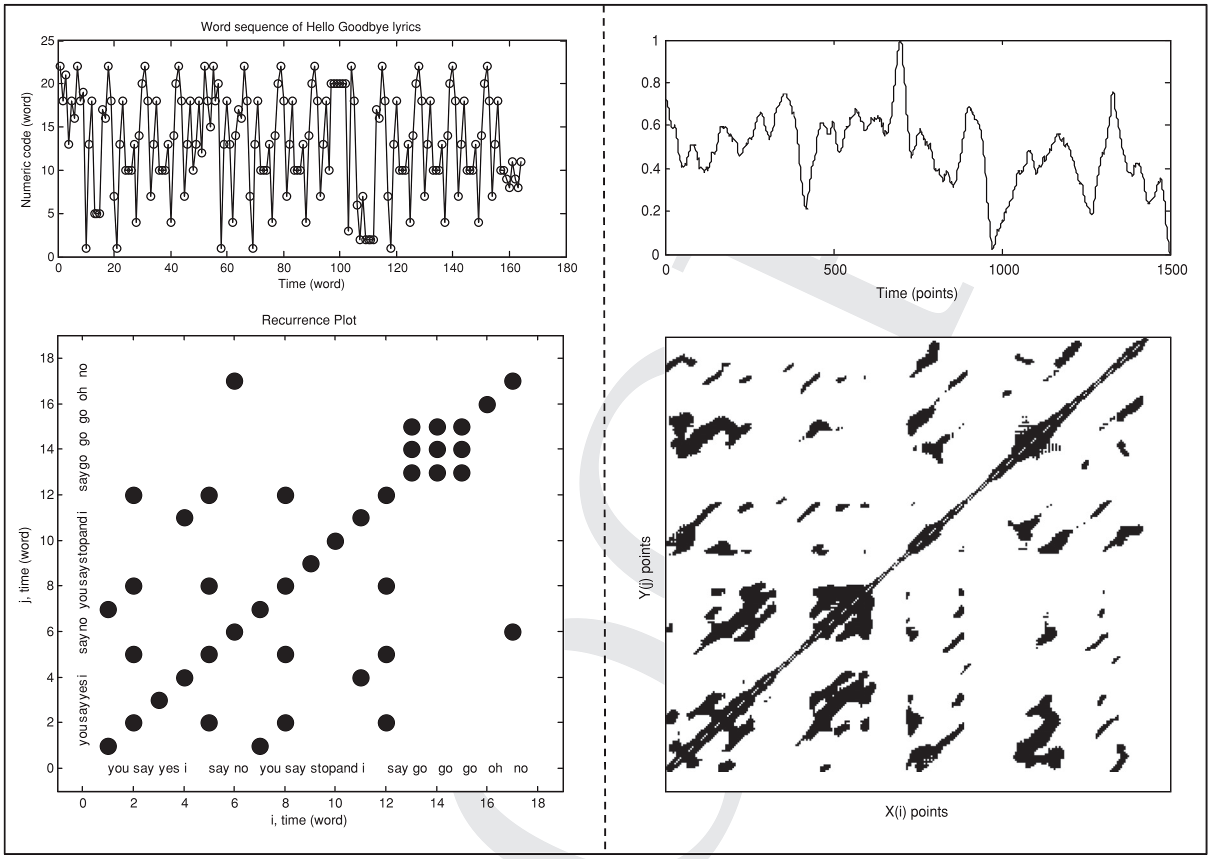


Figure 11.9. (top left) The full time series of words extracted from the lyrics of "Hello, Goodbye" by the Beatles. The y-axis represents the numeric identifier to which a word is assigned, and the x-axis represents word-by-word unfolding of this "lexical" time series. (bottom left) A recurrence plot of the first 20 words in the lyrics. Each point on the plot represent a relative point (*i,j*) in the lyrics at which a word is recurring. The "go go go" usage appears as a particular "texture" on the plot, along with the two-word sequence "you say" appearing as a diagonal line structure. (top right) The anterior-posterior postural sway movements of single individual standing and listening to another person speak for 30 seconds, recorded at 50 samples a second. (bottom right) A recurrence plot of the first 10 seconds of postural data. Despite the nonperiodic and nonstationary pattern of the postural sway movement, the recurrence plot reveals a significant degree of recurrent and deterministic structure, with the patterns of postural behavior reoccurring over time.

applicable methods for assessing the dynamics of biological and human behavior (e.g., Marwan & Meinke, 2002; Zbilut, Thomasson, & Webber, 2002), including social behavior (e.g., Dale & Spivey, 2005; Richardson et al., 2008; ; Shockley et al., 2003). Essentially, recurrence analysis identifies the dynamics of a system by discerning (1) whether the states of the system behavior recur over time and, if states are recurrent over time, (2) the degree to which the patterning of recurrences are highly regular or repetitive (i.e., deterministic). Conceptually, performing recurrence analysis on behavioral data is relatively easy to understand; one simply plots whether the recorded points, states, events, or categories in a time series or behavioral trajectory are revisited or reoccur over time on a two-dimensional plot, called a recurrence plot. This plot provides a visualization of the patterns of revisitations in a system's behavioral state space and can be quantified in various ways – a process known as recurrence quantification – in order to identify the structure of the dynamics that exist (see Marwan, 2008 and ; Weber & Zbliut, 2005 for more detailed reviews). The plots in Figure 11.9 are examples of what recurrence plots look like for a categorical (left plot) and continuous (right plot) behavioral time series.

Like spectral analysis and autocorrelation, recurrence analysis can also be extended to uncover the dynamic similarity, mutual influence, or coordinated structure that exists between two different behavioral time series or sequences of behavioral events. This latter form of recurrence analysis is termed *cross-recurrence analysis* and is performed in much the same way as standard (auto) recurrence analysis. The key difference is that recurrent points in a cross-recurrence plot correspond to states, events, or categories in two time series or behavioral trajectories that are recurrent with each other. Cross-recurrence analysis can therefore be employed to capture and then quantify the co-occurring or coordination dynamics of two behavioral time series or discrete behavioral sequences. Accordingly, some researchers have adopted cross-recurrence analysis to investigate semantic similarity in conversation (Angus, Smith, & Wiles, 2011), perceptual-motor synchrony between people interacting (Shockley et al., 2003; Richardson & Dale, 2005), and vocal dynamics during development (Warlaumont et al., 2010).

Fractal analysis. Researchers in social-personality psychology (or in any other field of psychology) commonly collapse repeated measurements into summary variables, such as the mean and standard deviation, under the assumption that the measured data contains uncorrelated variance or random fluctuations that are normally distributed. With respect to dynamic behavioral time-series data, however, this is rarely true, and thus summary statistics such as the mean and standard deviation often reveal little about how a system evolves over time. Indeed, time-series recordings of human performance and behavior typically contain various levels of correlated variance or data fluctuations (i.e., nonrandom fluctuations) that are not normally distributed (Stephen & Mirman, 2010) and, moreover, are structured in a *fractal* or *self-similar* manner (Gilden, 2001, 2009; Van Orden, Holden, & Turvey, 2003; Van Orden, Kloos, & Wallot, 2011).

A fractal or self-similar pattern is simply a pattern that is composed of copies of itself nested within itself. As a result, the structure looks similar at different scales of observation (i.e., magnification). Conceptually similar to geometric fractal patterns (Mandelbrot, 1982), a fractal time series is therefore a time series that contains nested patterns of variability (see Figure 11.10). That is, the patterns of fluctuation and change over time look similar at different scales of magnification or measurement resolution (i.e., as one zooms in and out).[5] The self-esteem time series in Figure 11.8 is a good example of a fractal or self-similar time-series pattern. This time series is displayed again in Figure 11.10, with the self-similarity of its temporal fluctuations revealed by zooming in on smaller and smaller sections. At each level of magnification the temporal pattern looks similar (see Bassingthwaighte, Liebovitch, & West, 1994 or Holden, 2005 for a more detailed tutorial).

A fractal time-series pattern is characterized by an inverse proportional relationship between the power (P) and frequency (f) of observed variation in a time series of measurements.[6] That is, for a fractal time-series, there exists a proportional relationship between the size of a change and how frequently changes of that size occur, with this relationship remaining stable across changes in scale. It is in this sense that the pattern of variability in a repeatedly measured behavior is self-similar; large-scale changes occur with the same *relative* frequency as small-scale changes. The degree to which a dataset approximates this ideal relationship between power and frequency, $P = 1/f^{\alpha}$, is summarized in the scaling exponent, α, with P = power, and f = frequency. If one plots the power of the different spectral frequencies that make up a time series on double-logarithmic axes, α is equivalent to the slope of the line that best fits the data (see Figure 11.11). That is, α captures the relationship between size and frequency of fluctuations in the time series of behavior. Random fluctuations (i.e., white noise) produce a flat line in a log-log spectral plot with a slope close to 0, which indicates that changes of all different sizes occur with approximately the same frequency in the time series. Alternatively, fractal fluctuations, often referred to as pink or $1/f$ noise, produce a line in a log-log spectral plot that has a slope closer to –1, which indicates the self-similar and scale-invariant scaling relationship characteristic of fractal patterns.

It is becoming increasingly clear that the behavior of most natural systems, including human and social systems, exhibit varying degrees of fractal structure (Delignières et al., 2006; Gilden, 2009; Holden, 2005). Moreover, the degree to which the fluctuations in a behavioral time series are fractal (i.e., pink) or not (i.e., white) provides evidence that a system

[5] In actuality, only ideal mathematical or geometric fractals are truly self-similar, with real-world fractals considered self-similar in a statistical sense. *Statistical self-similarity* simply means that a pattern is composed of statistically similar copies of itself (looks, on average, similar at different scales of observation).

[6] For an introductory review of the various methods that can be employed to measures the fractal structure of time-series data, see Delignières et al. (2006).

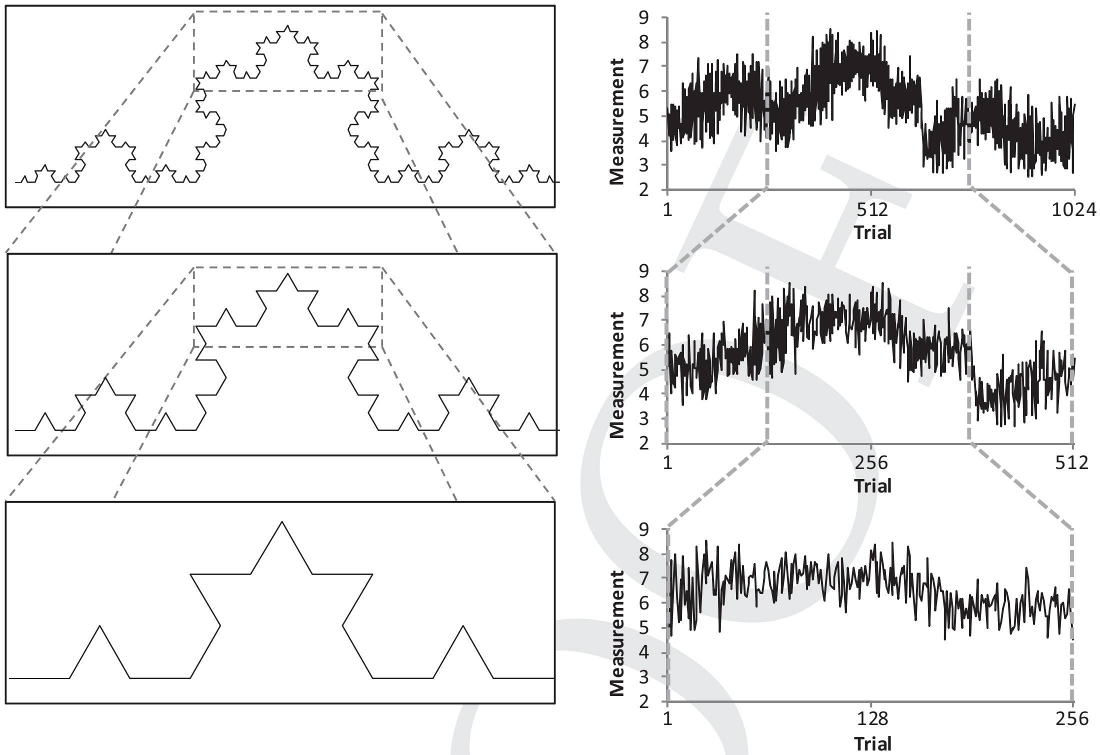


Figure 11.10. Example geometric and temporal fractal patterns. (left) Koch Snowflake at three levels of magnification. (right) The repeated self-esteem measurements presented in Figure 11.9 at three levels of magnification. The factual nature of these patterns is revealed by self-similar patterns being observed at smaller and larger magnitudes of observation (adapted from Holden, 2005).

is nonlinear and that its behavior is a consequence of interaction-dominant dynamics (Van Orden et al., 2003). To this extent, fractal patterns of behavior can also be a sign of emergence and self-organization. Although these ideas may seem foreign and irrelevant to the uninitiated, these analyses of fractal fluctuations have been applied fruitfully in the social domain. For example, the behavioral waves or periodic flow of social interaction has a fractal structure (Newtson, 1994), as do the dynamics of self-esteem (Delignières et al., 2004). More recently, Correll (2008) has shown that participants who are trying to avoid racial bias show a lesser fractal signature in their response latencies in a video game. Correll discusses these findings in light of characterizing social perception and other processes as a system of many intertwined dependencies – as processes of a complex dynamical system. So the behavioral fluctuations a person gives off may hint at social judgment events, such as stereotyping or racial bias. This avenue of research still seems relatively unexplored, and surprisingly so, if Correll's (2008) results are robust in multiple contexts.

Further Reading

Finally, we should note that there are a number of reviews of this material that can be consulted in social psychology (e.g., Nowak & Vallacher, 1998; Vallacher & Nowak, 1994) or within cognitive science more broadly (e.g., Port & Van Gelder, 1995; Spivey, 2007; Warren, 2006). Note as well that there are, of course, different levels of theoretical commitment to this agenda. For example, many dynamical systems approaches explicitly avoid the use of mental representations, and instead focus on perception-action couplings as a basis for understanding social and human behavior (e.g., Marsh et al., 2009; Richardson et al., 2009). Others focus more on the dynamics of internal mental processes or cognitive dynamics (e.g., Spivey, 2007). This leads to theoretical subtleties that cannot be conveyed here. If readers are intrigued to look further, we would encourage

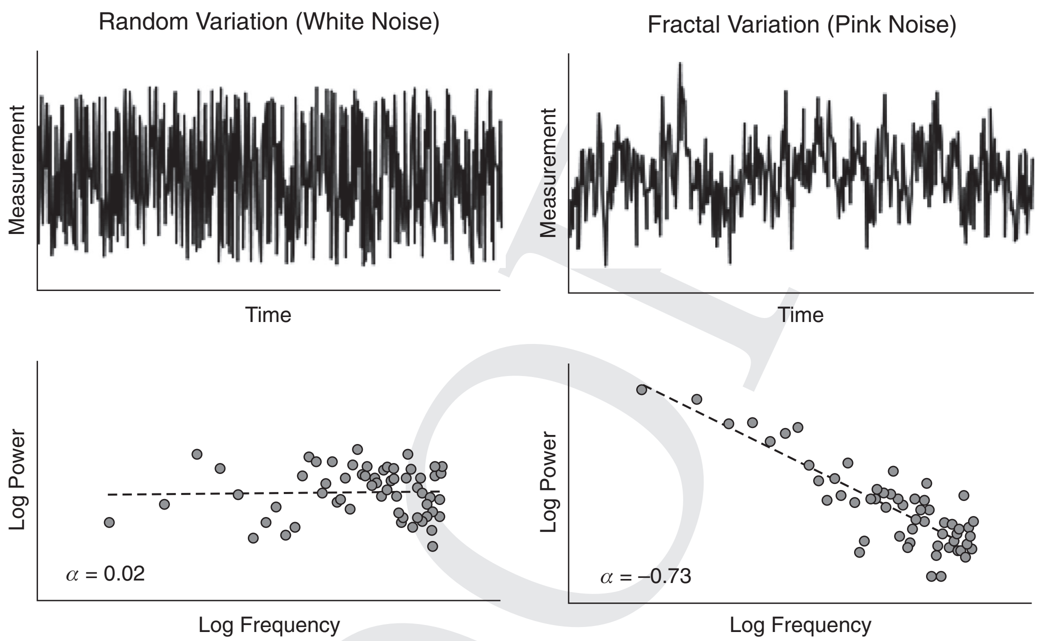


Figure 11.11. Examples of time series composed of random variation (top left) and fractal variation (top right) and the associated spectral plots with logarithmic axes (bottom left and right, respectively).

them to decide whether the theory, modeling, or analysis of dynamical systems motivates their interest. If it is deeply theoretical, for example, then much of the work we review would help tease apart those theoretical nuances. We also encourage readers to read the books of Kelso (1995), Nowak and Vallacher (1998), and Thelen and Smith (1994) for a more thorough, but still easy-to-read, introduction to dynamical systems theory and practice. If one is more interested in mathematical modeling, a review of the primary modeling papers cited will help the researcher jump straight into the concepts and their practicality in modeling. Reviews by Kaplan and Glass (1995) and Strogatz (1994) also provide a good introduction to the mathematical details of dynamical systems modeling. With respect to dynamical systems analysis, direct consultation with research that has previously employed a method of interest is always the best place to start. Throughout this chapter we have therefore included a range of relevant research articles that will enable any interested reader to gain a foothold on the relevant literature.

CONCLUSION

We had three key goals in this chapter. The first was to lay out some of the basic concepts behind complex dynamical systems. These concepts motivate theorists as they seek to understand social and cognitive systems as systems sustained by self-organization, bringing about soft-assembled processes, through nonlinear interaction-dominant dynamics. Our second key goal was to demonstrate how the dynamics of social processes and behavior can be explored directly using mathematical models. By laying out some of the mathematical modeling techniques that can be employed to understand dynamical systems, we showcased how the relatively new concepts of dynamical systems gain concrete manifestation in these explicit models. Our third goal was to provide a brief description of how the dynamics of behavior can be explored via the dynamical analyses of recorded data. Thus, in the third section of this chapter we described just a few of the many linear and nonlinear time-series analyses techniques that a social researcher could employ to investigate the dynamics inherent to his or her own behavioral (time-series) data.

Collectively, these sections urge social-personality psychologists to think of behavior as something that continually *changes* and, therefore, that must be studied and modeled as *time-evolving.* It is often challenging for a researcher to conceptualize his or her context of study in such a way that time series can be collected (see, for example, Correll's 2008 clever use of video games), or to adapt the perhaps unfamiliar concepts of self-organization or soft-assembly to their theories. Doing so, however, can help us understand the *processes* by which social behaviors come about in day-to-day activities and, thus, the approach will no doubt pay significant dividends for researchers interested in unveiling new domains of inquiry.

Acknowledgments

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