## (1)UCMERCED

# Introduction to Recurrence Quantification Analysis (RQA) 

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Van Orden, Holden, \& Turvey, 2003



## Linguistic Levels...



## Higher-Order States

- Interaction-dominant dynamics of the cognitive system produce higher-order collective properties that can be subjected to nonlinear methods.
- E.g.: behavior sequences (eyemovement fixations, actions, etc.)
- E.g.: linguistic sequences (words, sentences, etc.)



## Outline

- Time series of higher-order states - Analysis of series of behavioral categories
- The recurrence plot (RP) and "textures"
- Quantifying the plot (RQA)
- Examples and exercises



## Conversion...

- Converting transcripts to sequence of numeric identifiers:
- blue suede shoes ...
$b=1, l=2, u=3, e=4, \ldots$
-1234563474568946...
- Level of analysis here: letters



## Trajectories

- Think of the behavior sequences in geometric terms: trajectories in "category space."
- How repetitive is a sequence? If repetitive, how long are the repetitions? Lots of short bursts, or fewer but lengthy repeated trajectories?


## Recurrence Plot (RP)

- Bird's-eye view of the system's trajectories through its behavior space.




## Under the Hood

- If time series is:

$$
\mathbf{x}=1,3,2,3, \ldots, x_{N}
$$

- $\mathrm{RP}=$ set of points $(i, j)$ such that:

$$
x_{i}-x_{j}=0
$$

- In other words, set of points such that the numeric identifiers have a distance of zero from each other.


## Under the Hood

- In other words, the system's trajectory is revisiting the same state, or recurring in time.



# Exercise: Build Some RPs 



## Quantification

- You can eyeball a plot, but in real contexts we want some quantification so that plots (or, e.g., conditions) can be compared.
- Enter: recurrence quantification analysis (RQA).
- Quantities capturing the amount and distribution of points on the plot.


## RQA Measures

Recurrence rate (\%REC): Total percentage of the plot occupied by points.

$$
R R=\frac{1}{N^{2}} \sum_{i, j=1}^{N} \mathbf{R}_{i, j}
$$

$\underline{\text { http://www.recurrence-plot.tk }}$


## RQA Measures

Average diagonal line length (MEANLINE): Average length of diagonal lines on the plot excluding the line of incidence (length $>1$ ).

$$
L=\frac{\sum_{l=l_{\min }}^{N} l P(l)}{\sum_{l=l_{\min }}^{N} P(l)}
$$

http://www.recurrence-plot.tk


## RQA Measures

Maximum line length (MAXLINE): The longest diagonal line on the plot (excluding the line of incidence).
$L_{\text {max }}=\max \left(\left\{l_{i} ; i=1 \ldots N_{l}\right\}\right)$ http://www.recurrence-plot.tk


## RQA Measures

Entropy (ENTROPY): The entropy of the distribution of diagonal lines on the plot (how much "disorder" is there in the sequences).



## Three Important Concepts

- Parameters important for later...
- embedding dimension,
- delay,
- radius
- These parameters will be crucial when we apply these RQA methods to continuous data (e.g., posture).


## Embedding Dimension

- Interpretation in categorical data: how many states must match in order to count it as a recurrence.
- In previous analysis, dimension $=1$
- What about 2? 3?
- Window, vector, sequence, etc.



It is possible to use dimension with categorical data, and the window size can be determined by theoretical or practical concerns (e.g., avoiding single-letter recurrence, as above).

## Delay

- With most categorical data (behavior sequences, linguistic sequences, etc.) temporal ordering should probably be preserved (delay =1).
- NB: Situation more complex with continuous data (tomorrow).


## Radius

- The distance between units (or windows if dimension $>1$ ) required in order to count $(i, j)$ as a recurrent point.
- With category data: radius $=0$.
- NB: Situation more complex with continuous data.


## But, with categories...

- With most data of nominal codes (e.g., letters, words, etc.), the following parameters often suffice:
- Embedding dimension: 1
- Delay: 1
- Radius: . 0001


## Exercise: Quantify Plots!

See instructions on paper...

## Coco \& Dale, 2014

- With Dr. Moreno Coco University of Edinburgh
- R package for categorical recurrence (adaptable for continuous recurrence)
- Basic (C)RQA measures
- Diagonalwise recurrence (this afternoon)
- Windowed recurrence measures

Nonuniformity in behavioral dynamics

Behavioral "modes"
Stable, but temporary, functional structures


## Cattail Down

by MeWithoutYou
headed east out of st. paul, we stopped for water. rested in the cemetery, watched the mississippi. running out of food stamps, found a bag along the footpath off highway 61 filled with what looked like marijuana. (don't worry mom, we left it there) hopped a grainrail out of pig's eye toward milwaukee, a deer between the tower and the tracks, saw right through us. said, "you don't know where you came from, you don't know where you're going, you think you're youyou don't know who you are, you're not you.

## Cattail Down by MeWithoutYou



Tme
"Windowed recurrence"

Obtain several new time series of recurrence measures as they change across windows

## Coco \& Dale, 2014

```
runcrqa( ts1,
```

runcrqa( ts1,

```
    ts2,
```

    ts2,
    ```
    ts2,
    par )
    par )
    par )
    # par = list of parameters
    # par = list of parameters
    # par = list of parameters
    par = list(
    par = list(
    par = list(
        type = 2, step = 1,
        type = 2, step = 1,
        type = 2, step = 1,
        windowsize = 50,
        windowsize = 50,
        windowsize = 50,
        lagwidth = 40,
        lagwidth = 40,
        lagwidth = 40,
        method = "window",
        method = "window",
        method = "window",
        datatype = "categorical",
        datatype = "categorical",
        datatype = "categorical",
        thrshd = 8 )
```

```
        thrshd = 8 )
```

```
        thrshd = 8 )
```

```



Dole, Wariaumont, 8 Richaritoon



\section*{Example Analyses}
- Cross-linguistic...
- ...stylistic complexity
- ...encoding differences
- Within-language
- ...text analysis
- ...topic dynamics

like Sam I am Sam Sam I am That Sam---am That Sam-l-am! lo not like that Sam-l-am Do you you like them Here or there? I would not like them here or there. I would not like them anywhere. do not like green eggs and ham. I do not like them, Sam-l-am Would you like them in a house? Would you like them with a mouse? I do not like them in a house. I do not like them with a mouse. I do not like them here or there. I do not like them anywhere. I do not like green eggs and ham. I do not like them, Sam-l-am. Would you eat them in a box? Would you eat them with a fox? Not in a box. Not with a fox. Not in a house. Not with a mouse. I would not eat them here or there. I would not eat them anywhere. I would not eat green eggs and ham. I do not like them, Sam-l-am. Would you? Could you? in a car? Eat them! Eat them! Here they are. I would not, could not, in a car You may like them. You will see. You may like them in a tree? d not in a tree. I would not, could not in a tree. Not in a car! You let me be. I do not like them in a box. I do not like them with a fox I do not like them in a house I do mot like them with a mouse I do not like them here or there. I do not like them anywhere. I do not like green eggs and ham. I do not like them, Sam-l-am. A train! A train! A train! A anywhere. I do not like green eggs and ham. I do not like them, Sam-l-am. A train! A train! A train! A would not, could not, in a box. I could not, would not, with a fox. I will not eat them with a mouse I will not eat them in a house. I will not eat them here or there. I will not eat them anywhere. I do not like them, Sam-l-am. Say! In the dark? Here in the dark! Would you, could you, in the dark? I would like them, Sam-l-am. Say! In the dark? Here in the dark! Would you, could you, in the dark? I would
not, could not, in the dark. Would you, could you, in the rain? I would not, could not, in the rain. Not not, could not, in the dark. Would you, could you, in the rain? I would not, could not, in the rain. in the dark. Not on a train, Not in a car, Not in a tree. I do not like them, Sam, you see. Not in a
house. Not in a box. Not with a mouse. Not with a fox. I will not eat them here or there. I do not lik house. Not in a box. Not with a mouse. Not with a fox. I will not eat them here or there. I do not them anywhere! You do not like green eggs and ham? I do not like them, Sam-l-am. Could you, would you, with a goat? I would not, could not. with a goat! Would you, could you, on a boat? I could not, would not, on a boat. I will not, will not, with a goat. I will not eat them in the rain. I win not eat them on a train. Not in the dark! Not in a tree! Not in a car! You let me like! I do with a mouse. I do hot like them here or there. I do not like them ANYWHERE! I do not like green eggs and ham! I do not like them, Sam-l-am. You do not like them. SO you say. Try them! Try them! And you may. Try them and you may I say. Sam! If you will let me be, I will try them. You will see. Say! I like green eaas and ham! I do!! I like them. Sam-I-am! And I would eat them in a boat! And I would eat them


\section*{Example Application}
- Difference in the morphosyntactic dynamics of languages around the world.
- E.g., languages with highly complex morphology (e.g., Yagua, ...) vs. very simple morphology (e.g., English, ...).
- Encode information differently
- \%DET \(\propto\) population (Lupyan \& Dale, 2010)


Analysis: 103 languages that use Roman alphabet and have translation in UN-UDHR

\section*{Example Application}
- Genre identification in educational data mining contexts.
- Do "textual dynamics" differ across history, science, etc. texts?
- Do these dynamic patterns correlate with accessibility, learning gains, etc.?


\section*{Example Application}
- Identifying topic changes in very large texts.
- E.g., the pattern of points will change dramatically over a "topic" boundary.
- In natural language processing, related to methods using "dotplot."

\section*{Document Dynamics}


\section*{Dynamic Discourse Analysis}
- Collection of methods, models, etc. related to dynamic analysis of transcripts.
- Standard data format for dynamic discourse analysis: by-word long form (BWLF).
- B(eo)W(u)LF


\begin{tabular}{|c|c|c|c|c|c|c|}
\hline canto lin & line word & charnum & social & affect & relativ & time \\
\hline 1 & 110 & 2 & 0 & 0 & 0 & 0 \\
\hline 1 & 1 praise & 6 & 100 & 100 & 0 & 0 \\
\hline 1 & 1 of & 2 & 0 & 0 & 0 & 0 \\
\hline 1 & 1 the & 3 & 0 & 0 & 0 & 0 \\
\hline 1 & 1 prowess & 7 & 0 & 0 & 0 & 0 \\
\hline 1 & 1 of & 2 & 0 & 0 & 0 & 0 \\
\hline 1 & 1 people & 6 & 100 & 0 & 0 & 0 \\
\hline , & 1 kings & 5 & 0 & 0 & 0 & 0 \\
\hline 1 & 2 of & 2 & 0 & 0 & 0 & 0 \\
\hline 1 & 2 spear & 5 & 0 & 0 & 0 & 0 \\
\hline 1 & 2 armed & 5 & 0 & 0 & 0 & 0 \\
\hline 1 & 2 danes & 5 & 0 & 0 & & 0 \\
\hline 1 & 2 in & 2 & 0 & 0 & 100 & \(\bigcirc\) \\
\hline 1 & 2 days & 4 & 0 & 0 & 100 & 100 \\
\hline 1 & 2 long & 4 & 0 & 0 & 100 & 100 \\
\hline 1 & 2 sped & 4 & 0 & 0 & 100 & 100 \\
\hline 1 & 3 we & 2 & 100 & 0 & 0 & 0 \\
\hline 1 & 3 have & 4 & 0 & 0 & 0 & 0 \\
\hline 1 & 3 heard & 5 & 100 & 0 & 0 & 0 \\
\hline 1 & 3 and & 3 & 0 & , & 0 & 0 \\
\hline 1 & 3 what & 4 & 0 & 0 & 0 & 0 \\
\hline 1 & 3 honor & 5 & 0 & 100 & 0 & 0 \\
\hline 1 & 3 the & 3 & 0 & 0 & 0 & 0 \\
\hline 1 & 3 athelings & 9 & 0 & 0 & 0 & 0 \\
\hline 1 & 3 won! & 3 & 0 & 100 & 0 & 0 \\
\hline 1 & 4 oft & 3 & 0 & 0 & 0 & 0 \\
\hline 1 & 4 scyld & 5 & 0 & 0 & 0 & 0 \\
\hline 1 & 4 the & 3. & 0 & , & 0 & 0 \\
\hline 1 & 4 scefing & 7 & 0 & , & 0 & 0 \\
\hline
\end{tabular}

\section*{Visualizing Beowulf with B(eo)W(u)LF}
```

RP
Cartesian product of time indices of noted events
affect term indices $=(1,4,423)$
$R P=$ affect terms $\times$ affect terms
$=[(1,1),(1,4),(1,423),(4,1),(4,4), \ldots]$
akin to Hasselman, 2009 (

```

Discursis (Angus et al.)

```

\& % D wwwdscursiscom

```

Discursis
моме
                            sur sestabci
                                    teav
                                    matsentations

\section*{Seeing the conversation}

Coessd at the Unversity of Cueendend, Austrata.
Discurnh is a compate- bewed tod toc ansuping human commenication Discuris can assas prectibenes in



\section*{Outline: Part 1}
- Time series of higher-order states - Analysis of series of behavioral categories
- The recurrence plot (RP) and "textures"
- Quantifying the plot (RQA)
- Examples

\section*{Additional Exercises...}
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